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Institute of Mathematics, National Academy of Sciences of Ukraine
Taras Shevchenko National University of Kyiv
University of Oslo
Vasyl' Stus Donetsk National University
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International Conference «Stochastic Equations, Limit Theorems and Statistics of Stochastic Processes»

dedicated to the 100th anniversary of I. I. Gikhman

September 17–22, 2018

ABSTRACTS

Igor Sikorsky Kyiv Polytechnic Institute
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International Conference «Stochastic Equations, Limit Theorems and Statistics of Stochastic Processes», dedicated to the 100th anniversary of I. I. Gikhman, September 17–22, 2018, Kyiv, Ukraine

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Contents

ON EXISTENCE AND UNIQUENESS OF A STRONG STATIONARY SOLUTION TO AN SDE WITH NON-REGULAR DRIFT

OLGA ARYASOVA, ANDREY PILIPENKO

Consider a d-dimensional stochastic differential equation

$$\begin{cases} d\varphi_t(x) = (-\lambda\varphi_t(x) + \alpha(\varphi_t(x)))\, dt + \sum_{k=1}^{m} \sigma_k(\varphi_t(x)) dw_k(t), t \geq 0, \\ \varphi_0(x) = x, \end{cases} \tag{1}$$

where $x = (x^1, \ldots, x^d) \in \mathbb{R}^d$, $\lambda > 0$, $(w(t))_{t\geq 0} = (w_1(t), \ldots, w_m(t))_{t\geq 0}$ is a standard m-dimensional Wiener process, $\alpha : \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma = (\sigma_1, \ldots, \sigma_m) : \mathbb{R}^d \to \mathbb{R}^d \times \mathbb{R}^m$ are bounded functions.

Assume that

(1) σ satisfies the Lipschitz condition and the uniform ellipticity condition,
(2) $\alpha(x) = \alpha_+(x)1\!\!1_{x\in\mathbb{R}^d_+} + \alpha_-(x)1\!\!1_{x\in\mathbb{R}^d_-}$, where $\alpha_\pm$ are Lipschitz functions, i.e., the drift has a jump discontinuity along a hyperplane $\mathbb{R}^{d-1}\times\{0\}$ and is Lipschitz continuous in the upper and the lower half-spaces.

We give sufficient conditions that ensure $|\varphi_t(y) - \varphi_t(x)| \to 0,\ t \to \infty$, convergence to zero of the distance between solutions that started from different points.

We also prove existence and uniqueness of a strictly stationary strong solution to (1).

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ABOUT ONE GOODNESS-OF-FIT TEST

PETRE BABILUA, ELIZBAR NADARAYA

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent, equally distributed random variables, having a distribution density $f(x)$. Based on sample $X_1, X_2, \ldots, X_n$ it is required to check the hypothesis

$$H_0 : f(x) = f_0(x).$$

Here we consider the hypothesis H_0 testing, based on the statistics

$$T_n = n a_n^{-1} \int (f_n(x) - f_0(x))^2 r(x)\, dx,$$

where $f_n(x)$ is the recurrent Wolverton–Wagner kernel estimate of probability density defined by:

$$f_n(x) = n^{-1} \sum_{i=1}^{n} a_i K((a_i(x - X_i))),$$

where a_i is an increasing sequence of positive numbers tending to infinity, $K(x)$, $f_0(x)$ and $r(x)$ satisfy certain regularity conditions.

- Question of consistency for the constructed criterion against any alternative $H_1 : f(x) = f_1(x)$, where $f_1(x)$ is such that $\int (f_n(x) - f_0(x))^2 r(x)\, dx > 0$ is studied.
- The limiting behavior of the power is studied for sequence of close to hypothesis H_0 alternatives of type Pitmen and Rosenblatt [1] and it is shown that the tests based on T_n for above mentioned alternatives are more powerfull in limits than the tests based of Bickel–Rosenblatt [2].

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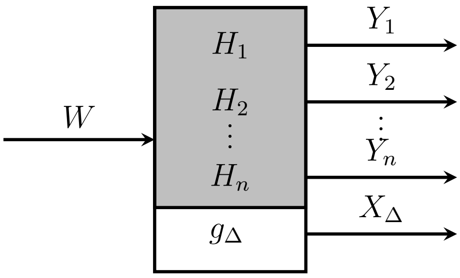
PROPERTIES OF CORRELOGRAM-BASED ESTIMATORS IN SIMO-SYSTEMS

I. BLAZHIEVSKA, V. ZAIATS

A lot of problems in different branches of engineering and science (signal and image processing, communications and networks, control), finance, medicine and biology may be solved under an appropriate choice from the approximating models. In the talk, we consider a "black–box" model (which admits some input and output signals) and cover the problem how to estimate its parameters. The random character and the "independence of past time moments and places of simulation" of the signals lead us to stochastic linear time-invariant systems (LTI) with stationary inputs. Notice the important fact: any LTI system is uniquely identified by means of an impulse response function (IRF). We demonstrate this feature on a single-input multiple-output (SIMO) channel-model.

Let us be more specific. Consider the following LTI SIMO system whose IRF has $(n+1)$ real-valued and $L_2(\mathbf{R})$-integrable components (or kernels):



Here, $H_j = (H_j(\tau), \tau \in \mathbf{R}), j = 1, ..., n$, are n unknown functions while $g_\Delta = (g_\Delta(\tau), \tau \in \mathbf{R})$, is a known function depending on a parameter $\Delta > 0$ and having a δ-like structure as $\Delta \to \infty$. We shadow the channels where unknown IRF components appear.

System's input is a standard Wiener process $W = (W(t), t \in \mathbf{R})$, whereas outputs are:

$$Y_j(t) = \int_{-\infty}^{\infty} H_j(t-s)dW(s), \quad j = 1, 2, ..., n, \quad t \in \mathbf{R};$$

$$X_\Delta(t) = \int_{-\infty}^{\infty} g_\Delta(t-s)dW(s), \quad t \in \mathbf{R}.$$

Above integrals are called Wiener shot noise processes; they are jointly Gaussian, stationary, zero-mean processes having spectral densities [3].

After observation of the outputs $Y_1, Y_2, ..., Y_n$ and X_Δ, it is possible to consider three statistical problems:

- to estimate the unknown correlation function of each $Y_j, j = 1, 2, ..., n$;
- to estimate the unknown cross-correlation function of any pair Y_j and Y_k, $j \neq k$;
- to estimate the unknown kernels $H_j, j = 1, 2, ..., n$.

The approach solving all of these problems is based on constructing integral-type correlograms between the pairs of outputs belonging to three classes, $\{Y_j, Y_j\}$, $\{Y_j, Y_k\}$ and

$\{Y_j, X_\Delta\}$ respectively. After some adaptations, the solutions of the first two problems may be taken from [3, 4]; more precise, the results on asymptotic normality and on constructing of confidence intervals for the corresponding estimators in different functional spaces may be found there. The third problem covers the both above since correlation and cross-correlation functions are identified in terms of convolutions between the pairs of kernels H_j $(j = 1, 2, ..., n)$ only. That is why we pay an attention to the last problem only.

For any $j = 1, 2, ..., n$, we use the following scaled sample cross-correlogram between Y_j and X_Δ as an estimator for H_j:

$$\widehat{H}_{j,T,\Delta}(\tau) = \frac{1}{T}\int_0^T Y_j(t+\tau)X_\Delta(t)dt, \quad \tau \in \mathbf{R},$$

where T is the length of the averaging interval. The fact that the estimator $\widehat{H}_{j,T,\Delta} = \big(\widehat{H}_{j,T,\Delta}(\tau), \tau \in \mathbf{R}\big)$ is biased and depends on two parameters, T and Δ, enable us to study the role of these parameters in obtaining nice statistical properties of the estimator.

Our investigation deals with asymptotic behaviour of the IRF component's estimator $\widehat{H}_{j,T,\Delta}$ as $T \to \infty$ and $\Delta \to \infty$ in different functional spaces. Depending on a space we choose, the basic assumptions $H_j \in L_2(\mathbf{R}), j = 1, 2, ..., n$, and $g_\Delta \in L_2(\mathbf{R})$, are complemented with some extras related to sample behaviour of Gaussian processes and quadratic forms of Gaussian processes. For this purpose, we apply some techniques from [1]–[5]. We compare our results to earlier ones [6] and give illustrative examples [7].

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ASYMPTOTICS FOR WAVE EQUATIONS WITH STOCHASTIC MEASURES

IRYNA BODNARCHUK

Let $\mathcal{B}$ be the Borel σ-algebra of subsets of $[0,T]$, $T > 0$, $L_0(\Omega, \mathcal{F}, \mathsf{P})$ be the set of real-valued random variables defined on a complete probability space $(\Omega, \mathcal{F}, \mathsf{P})$.

Consider the Cauchy problem for the stochastic wave equation

$$\begin{cases} \dfrac{\partial^2 u(t,x)}{\partial t^2} = a^2 \Delta_x u(t,x) + f(t,x,u(t,x)) + \sigma(t,x)\,\dot{\mu}(t), \\ u(0,x) = u_0(x); \quad \dfrac{\partial u(0,x)}{\partial t} = v_0(x), \end{cases} \tag{1}$$

where $(t,x) \in [0,T] \times \mathbb{R}^d$, $d = 1,2$, $a > 0$, Δ_x is Laplace operator. Here μ is a general stochastic measure defined on $\mathcal{B}$, i.e. $\mu : \mathcal{B} \to L_0(\Omega, \mathcal{F}, \mathsf{P})$ is a σ-additive mapping.

We investigate the mild solution of (1), i. e., any measurable random function $u(t,x) = u(t,x,\omega) : [0,T] \times \mathbb{R}^d \times \Omega \to \mathbb{R}$ such that $\forall\,(t,x)$:

$$\begin{aligned} u(t,x) = &\int_{\mathbb{R}^d} S_d(t,x-y) v_0(y)\,dy + \frac{\partial}{\partial t}\left(\int_{\mathbb{R}^d} S_d(t,x-y) u_0(y)\,dy\right) \\ &+ \int_0^t ds \int_{\mathbb{R}^d} S_d(t-s,x-y) f(s,y,u(s,y))\,dy \\ &+ \int_{(0,\,t]} d\mu(s) \int_{\mathbb{R}^d} S_d(t-s,x-y)\sigma(s,y)\,dy\,. \end{aligned}$$

Here $S_d(t,x)$ is the fundamental solution of the wave equation.

We assume some conditions on functions v_0, u_0, σ, f like measurability, boundedness and regularity.

The existence, uniqueness and Hölder continuity of the mild solution are proved in [1] (one-dimensional case) and in [2] (two-dimensional case).

We investigate the asymptotic behavior of the mild solution as the spatial variable tends to infinity.

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THE MODIFIED EULER SCHEME FOR A WEAK APPROXIMATION OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN BY A WIENER PROCESS

SEMEN BODNARCHUK

Our goal is to provide direct and clear way for obtaining the weak approximation of diffusion processes of the form

$$X_t = x + \int_0^t a(X_s)ds + \int_0^t \sigma(X_s)dW_s, \quad 0 \le t \le T,$$

where $W = (W^1, \dots, W^m)$ is a Wiener process, $x \in \mathbb{R}^d$, $a : \mathbb{R}^d \to \mathbb{R}^d$, $\sigma : \mathbb{R}^d \to \mathbb{R}^{d\times m}$. In the classical approach the Ito-Taylor expansion is used for such a reason (see, for example, book [1]). But in this case we need to simulate multiple Ito integrals which is quite complicated problem. Instead of such multiple integrals the random variables which have to satisfy some moment conditions can be used (for details, see [1], Corollary 5.12.1). And it is not so convenient, because in multi-dimensional case for weak approximation of higher order the choosing of such variables is not very clear. We propose another way for obtaining the weak approximation of diffusion processes which avoids all mentioned above difficulties.

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LARGE DEVIATIONS FOR RANDOM WALK IN RANDOM ENVIRONMENT

DARIUSZ BURACZEWSKI, PIOTR DYSZEWSKI

In this talks we will discuss recent results concerning large deviations of one-dimensional nearest neighbour random walk in site-random environment. Given a random environment $\omega = (\omega_i)_{i\in\mathbb{Z}}$ consisting of random weights from (0,1) we define random walk on $\mathbb{Z}$

$$\mathbb{P}_\omega(X_{n+1} = j | X_n = i) = \begin{cases} \omega_i & \text{if } j = i+1, \\ 1-\omega_i & \text{if } j = i-1, \\ 0 & \text{otherwise.} \end{cases}$$

Our main contribution is an extension of large deviation results for (X_n) to precise (rather than logarithmic) asymptotic. We will explain how this result is related to large deviations of branching process in random environment and stochastic recurrence equations

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STOCHASTIC SYSTEMS WITH MEMORY, ROBUSTNESS AND SENSITIVITY

GIULIA DI NUNNO

Stochastic systems with memory naturally appear in life science, economy, and finance. We take the modelling point of view of stochastic functional delay equations and we study these structures. We study the case when the driving noises admit jumps providing results on existence and uniqueness of strong solutions, estimates for the moments and the fundamental tools of calculus. We study the robustness of the solution to the change of noises. Specifically, we consider the noises with infinite activity jumps versus an adequately corrected Gaussian noise.

In the case of Brownian driving noise, we consider evaluations based on these models (e.g. the prices of some financial products) and the risks connected to the choice of these models. In particular we focus on the impact of the initial condition on the evaluations. This problem is known as the analysis of sensitivity to the initial condition and, in the terminology of finance, it is referred to as the Delta. In this work the initial condition is represented by the relevant past history of the stochastic functional differential equation. This naturally leads to the redesign of the definition of Delta. We suggest to define it as a functional directional derivative, this is a natural choice. For this we study a representation formula which allows for its computation without requiring that the evaluation functional is differentiable. This feature is particularly relevant for applications.

Our techniques make use of stochastic calculus via regularisations, Malliavin/Skorohod calculus and functional derivatives.

The presentation is based on joint works with: D.R. Banos, F. Cordoni, L. Di Persio, H.H. Haferkorn, F. Proske, E.E. Røse. See [1], [2].

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OPEN DYNAMIC SYSTEMS. CONSERVATION LAWS AND HARMONIZATION

VALERIY A. DOOBKO

1. Numerous problems of real systems study can be solved by modelling their evolution on the basis of equations of dynamic system influenced by random perturbation [1]-[4]. In making up these equations, concrete assumptions about preserved functionals, which are determined by the nature this phenomenon are being used. If a class of equations is singled out properly, functional-analogs from dynamic variables must become stable characteristics of the system. On the other hand, singling out preserved functionals for the given set of equations, can serve as an indication of their existence in a modelling system. The search for these functionals is the main goal of modelling, as well as establishing dynamics of transition between them. For example, given the system of equations reflect the evolution of the system of particles, which don't disappear and are not multiplied by others, the number of particles will be the example the preserved functional. For the system of equations, if the conditions of existence and uniqueness of the decision are fulfilled, we can affirm that the existence of such an invariant for the ensemble of stochastic systems. This is an integral of density for these systems. We considered some examples that demonstrate this general statement and methods of its realization: construction of solutions of definite class of stochastic differential equations in partial derivations; conservation of functional for the evolution in random environment, problem of existence [1], [4].

2. The Theories of self-organization and evolution are based on idea of a possibility the existence of synergetic phenomenon's for complexity multi-element systems.However, these theories It is not possible to explain the stable nature of the existence of a complex consisting of finite number of elements. It is usually assumed that the Self-organization effect can be is manifested only with an unlimited growth of a number of homo- and heterogeneous elements and their connections These conclusions allow us to exam some traditional considerations of Modern Biology in new aspects. When studying this problem we were come to following conclusion. Under conditions of strong and noncausal interactions with by environment there are a possibility of causal changes of characteristics in the hierarchic organization's complexity multi-element systems and for systems that have limited number of elements. As emphasized above this point of view is essentially different as compared with approaches, which are used for explanation a phenomenon of self-organization in the stochastic system. Above mentioned problems as we believe, require their solution [2]. We note that the problems of modelling open systems required the development of the theory of stochastic differential equations [5]. A theory of integral invariants, first integrals, is created. The Ito-Ventzel formula was generalized for the Wiener and Poisson perturbations [1].

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ASYMPTOTICS OF INTERSECTION LOCAL TIME FOR CORRELATED BROWNIAN MOTIONS

ANDREY DOROGOVTSEV, OLGA IZYUMTSEVA

Consider a diffusion process $x(t),\ t > 0$ with the following infinitesimal generator

$$L = \nabla \cdot (A\nabla),$$

where

$$A = (a_{ij})_{ij=1}^{4} : \mathbb{R}^4 \to \mathbb{R}^{4\times 4}$$

is symmetric, positive definite, uniformly elliptic. Suppose that A is smooth with bounded derivatives of all orders. It is known [1] that under announced assumptions on the operator L the process x has a transition density $p.(\cdot,\cdot) \in C((0;+\infty)\times\mathbb{R}^4\times\mathbb{R}^4)$ which satisfies the following upper and lower heat kernel estimates. There exists $c \geq 1$ such that for any $t > 0,\ x, y \in \mathbb{R}^4$:

$$\frac{1}{ct^2}e^{-\frac{c\|x-y\|^2}{t}} \leq p_t(x,y) \leq \frac{c}{t^2}e^{-\frac{\|x-y\|^2}{ct}}.$$

In the talk we treat the two-coordinates components $x_1,\ x_2$ of x as an analog of dependent Brownian motions on the plane. The main object of our investigation is the intersection local time for $x_1,\ x_2$ which is formally defined by the formula

$$I_t^x = \int_{\Delta_2(t)} \delta_0(x_2(t_2) - x_1(t_1))d\vec{t}.$$

To give a precise meaning for I_t^x consider approximations

$$I_{t,\varepsilon}^x = \int_{\Delta_2(t)} f_\varepsilon(x_2(t_2) - x_1(t_1))d\vec{t}.$$

Here

$$\Delta_2(t) = \{0 \leq s_1 \leq s_2 \leq t\},$$

δ_0 is the delta-function at the point zero,

$$f_\varepsilon(y) = \frac{1}{2\pi\varepsilon}e^{-\frac{\|y\|^2}{2\varepsilon}},\ y \in \mathbb{R}^2,\ \varepsilon > 0.$$

It is naturally to try to define I_t^x as the limit in mean square of approximations $I_{t,\varepsilon}^x$ as $\varepsilon \to 0$.

Theorem 1. *For any $t > 0$ there exists the intersection local time*

$$I_t^x := L_2\text{-}\lim_{\varepsilon\to 0} I_{t,\varepsilon}^x.$$

Let $w_1(t),\ w_2(t),\ t > 0$ be the independent planar Brownian motions which can be considered as the two-coordinates components of Brownian motion $w(t),\ t > 0$ in $\mathbb{R}^4$. It was proved in [2] that there exists

$$I^w := L_2\text{-}\lim_{\varepsilon\to 0}\int_{\Delta_2(1)} f_\varepsilon(w_2(t_2) - w_1(t_1))d\vec{t}.$$

Lemma 1. *(Moment estimates) For any $m \in \mathbb{N}$*

$$E(I_t^x)^m \le (2\pi)^{2m}\ c^{5m}\ t^m\ E(I^w)^m$$

and

$$E(I_t^x)^m \ge \frac{(2\pi)^{2m}}{c^{5m}}\ t^m\ E(I^w)^m.$$

Lemma 2. *(Upper estimate) For any $\lambda > 0$*

$$\limsup_{t\to+\infty} \frac{1}{llnt}\ ln\ P\Big\{\frac{I_t^x}{t\ llnt} > \lambda\Big\} \le -\frac{2\lambda}{c^5\pi^2\kappa^4},$$

where κ is the best constant of Gagliardo-Nirenberg inequality, i.e.

$$\kappa = \inf\{c > 0;\ \|f\|_4 \le c\ \|\nabla f\|_2^{\frac12}\|f\|_2^{\frac12},\ f \in L_4(\mathbb{R}^2),\ \nabla f \in L_2(\mathbb{R}^2)\}.$$

Lemma 3. *(Lower estimate) Put $\kappa_0 = 4\ln\kappa$. For any $a > 0$*

$$\inf_{t>0} \frac{1}{a} \ln P\Big\{\frac{1}{c_1} I_t^x \frac{1}{t} > a\Big\} \ge -A,$$

where

$$A = (-2\kappa_0 - 1 - 2\varepsilon)e^{\kappa_0}e^{\varepsilon},\ \varepsilon > 0,\ c_1 > 0.$$

Theorem 2. *(The law of iterated logarithm)*

$$\limsup_{t\to+\infty} \frac{I_t^x}{t\ llnt} = c_0\ a.s.,$$

where $c_0 \in (0;+\infty)$.

Obtained asymptotics is the same as for two independent planar Brownian motions [2]. We also discuss a comparison with the asymptotics of winding number for planar Brownian motions in the isotropic stochastic flow [3–5].

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SIMULATION OF GAUSSIAN PROCESS WITH CORRELATION FUNCTION OF A SPECIAL FORM

OLEKSANDR DYKHOVYCHNYI, NATALIIA KRUGLOVA

Gaussian processes whose correlation function is of a special form arise in many problems of finding distribution of a functional of some stochastic process which is a restriction of two-dimensional Chentsov random field on some set. Among other questions, determining the exact distribution of functionals (in particular, their maximum) of such processes is of a particular interest. In general, analytical form of distribution of such a functional is unknown. Therefore, the problem of finding empirical distribution of the maximum needs to be solved by simulation of an appropriate random process whose correlation function is known.

A lot of well-known methods for simulating Gaussian random processes exist, e.g. Cholesky method, Levinsons method, Wood and Chans method, etc. [2]. However, these algorithms are quite generic. We suggest building a computationally efficient algorithm that would account for the specifics of correlation function of the process in question.

Let $Y(t)$, $t \in [0,1]$ be continuous Gaussian process with expectation $E[Y(t)] = 0$ and correlation function $R(s,t) = u(s)v(t), s \leq t$; such that $a(t) = u(t)/v(t)$ is strictly increasing and its inverse, $a^{-1}(t)$, exists.

According to Doob's Theorem [1], the processes $Y(t)$ and $v(t)w(a(t))$, where $w(t)$,$t \in [0,1]$ is a Wiener process, are stochastically equivalent.

Let $Y(t)$, $\forall t \in [0,1]$ be a process with correlation function as described above. Then, we can simulate a trajectory of the Gaussian process $\hat{Y}(t)$ as follows.

Let us divide $[0,1]$ into n equal paths. Denote $t_k = \frac{k}{n}$ and $\Delta a(t_k) = a(t_k) - a(t_{k-1})$, $k = \overline{0,n}$.

Algorithm 1. *Generate a random variable ξ_0 from the standard Gaussian distribution. Set $\hat{Y}(0) = v(0)\xi_0\sqrt{a(0)}$ and $i = 0$ and perform the following steps:*

(1) *i:=i+1;*
(2) *generate a random variable ξ_i from the standard Gaussian distribution;*
(3) *set $\hat{Y}(t_i) = v(t_i)\sum_{k=1}^{i}\sqrt{\Delta a(t_k)}\xi_k$;*
(4) *if $i \leq n$, return to step 1.*

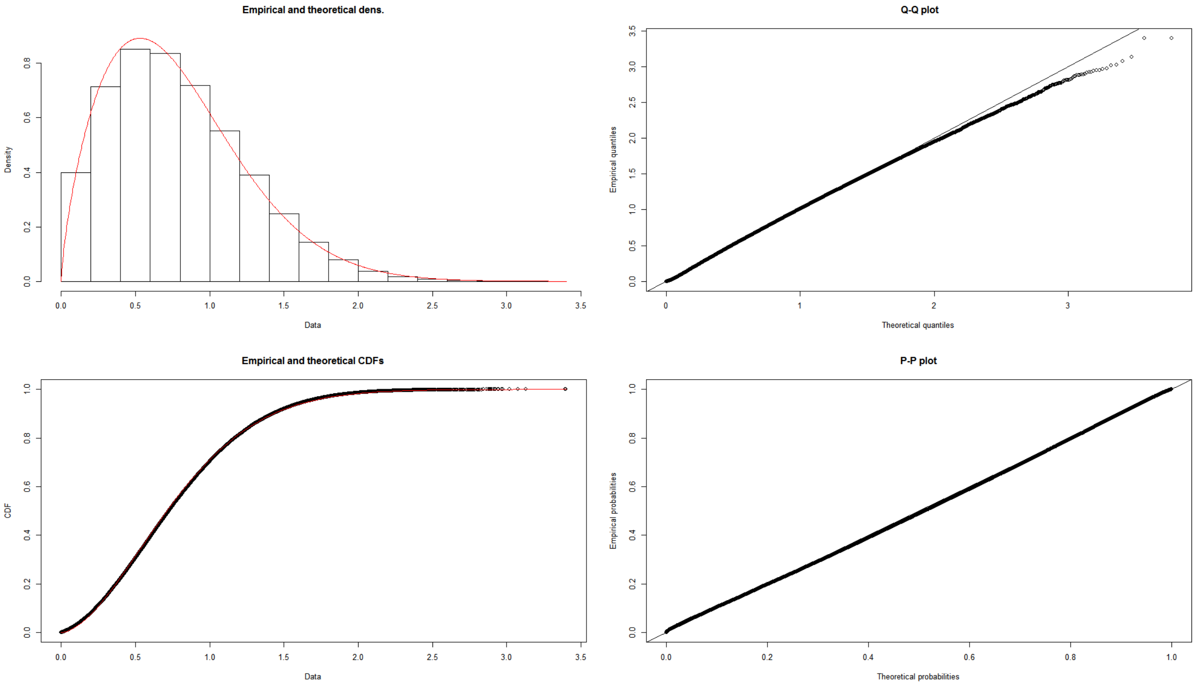
In this way we can simulate finite-dimensional distributions of the process $\hat{Y}(t_k)$, $k = \overline{0,n}$, whereas on intervals $(t_k,\, t_{k+1})$ $\hat{Y}(t)$ may be approximated with a linear interpolation.

Clearly, the algorithm can be easily implemented with R language.

Consider an example of applying the algorithm to determine distributions of a certain functionals.

Example 1. *Let us consider Chentsov random field $X(s,t)$ on the unit square and its restriction $Z(t) = X(t, \sqrt{1-t^2})$,$\forall t \in [0,1]$. We will construct simulation $\hat{Z}(t)$ of the process $Z(t)$ on the grid G: $0 = t_0 < t_1 < \ldots < t_{n-1} = \frac{n-1}{n}$, using $v(t) = \sqrt{1-t^2}$, $a(t) = \frac{t}{\sqrt{1-t^2}}$.*

We have found an empirical distribution for the maximum of this process. For this purpose 10^4 realizations of the process $\hat{Z}(t)$ were generated and $\max_{[0,1)} Z(t)$ were compared to known distributions. The closest distribution found turned out to be Weibull distribution. This was also verified with Kolmorgov-Smirnov test. The picture below shows histograms and empirical CDFs for both obtained distribution and Weibull distribution as well as Q-Q and P-P plots illustrating their proximity.



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A TRANSFORMED STOCHASTIC EULER SCHEME FOR MULTIDIMENSIONAL TRANSMISSION PDE

PIERRE ÉTORÉ, MIGUEL MARTINEZ

In this paper ([1]) we consider multi-dimensional partial differential equations of parabolic type involving divergence form operators that possess a discontinuous coefficient matrix along some smooth interface. The solution of the equation is assumed to present a compatibility transmission condition of its conormal derivatives at this interface (multi-dimensional diffraction problem, studied in [2]; the problem is also connected, on the stochastic aspect, to [3]). We prove an existence and uniqueness result for the solution and construct a low complexity numerical Monte Carlo stochastic Euler scheme to approximate the solution of the parabolic partial differential equation in divergence form. In particular, we give new estimates for the partial derivatives of the solution. Using these estimates, we prove a convergence rate for our stochastic numerical method when the initial condition belongs to an iterated domain of the divergence form operator. Our method presents the same convergence rate as the stochastic numerical schemes elaborated for the same problem in the one-dimensional context ([4]). Finally, we compare our results to classical deterministic numerical approximations and illustrate the accuracy of our method.

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THE ABSOLUTE CONTINUITY OF THE DISTRIBUTIONS OF SYSTEMS OF INTERACTING BROWNIAN PARTICLES

VLADIMIR FOMICHOV

Definition. A random field $\{x(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ is called a *Harris flow* with covariance function Γ if it satisfies the following conditions:

(i) for any $u \in \mathbb{R}$ the stochastic process $\{x(u,t),\ t \geqslant 0\}$ is a Brownian motion with respect to the common filtration $(\mathcal{F}_t := \sigma\{x(v,s),\ v \in \mathbb{R},\ 0 \leqslant s \leqslant t\})_{t\geqslant 0}$ and $x(u,0) = u$;

(ii) for any $u, v \in \mathbb{R}$ from $u \leqslant v$ it follows that $x(u,t) \leqslant x(v,t)$ for all $t \geqslant 0$;

(iii) for any $u, v \in \mathbb{R}$ the joint quadratic variation of the martingales $\{x(u,t),\ t \geqslant 0\}$ and $\{x(v,t),\ t \geqslant 0\}$ is given by

$$\langle x(u,\cdot), x(v,\cdot)\rangle_t = \int_0^t \Gamma(x(u,s) - x(v,s))\, ds, \quad t \geqslant 0.$$

The existence of such stochastic flows under mild assumptions on the covariance function Γ was proved in [2]. If $\Gamma = 1\!\!\mathrm{I}_{\{0\}}$, the corresponding stochastic flow is called the Arratia flow. Informally speaking, the Arratia flow is a system of Brownian particles starting from every point of the real line, any two of which move independently until they collide and after that coalesce and move together. This informal interpretation allows to construct the Arratia flow with an arbitrary drift satisfying the Lipschitz condition and to prove the Girsanov theorem for such stochastic flows (see [1]).

The structure of the dependence between particles in Harris flows makes the construction of Harris flows with drift much less straightforward. In our talk we will discuss the definition of Harris flows with drift and prove their existence. We will also describe the set of admissible shifts for the distributions of the n-point motions of Harris flows and prove the corresponding Girsanov theorem.

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ASYMPTOTIC PROPERTIES OF THE NUMBER OF CLUSTERS IN A ONE-DIMENSIONAL SYSTEM OF BROWNIAN PARTICLES

E. V. GLINYANAYA

We consider a one-dimensional system of Brownian particles with interaction.

Definition 1 ([1]). The Harris flow with the local characteristic Γ is a family $\{x(u,\cdot),\ u \in \mathbb{R}\}$ of Brownian martingales with respect to the joint filtration such that:
1) for every $u \in \mathbb{R}$ $\ x(u,0) = u$;
2) for every $u_1,\ u_2 \in \mathbb{R},\ u_1 \le u_2,\ t \ge 0:\ x(u_1,t) \le x(u_2,t)$;
3) for every $u_1,\ u_2 \in \mathbb{R}$ the joint quadratic variation of $x(u_1,\cdot)$ and $x(u_2,\cdot)$ is given by $d\left\langle x(u_1,\cdot), x(u_2,\cdot)\right\rangle(t) = \Gamma(x(u_1,t) - x(u_2,t))dt$.

Remark. In [1] the existence of x is proved for a real continuous positive definite function such that $\Gamma(0) = 1$ and Γ is Lipshits outside any neighborhood of zero. In the case when $\Gamma = \mathbb{I}_{\{0\}}$ existence of x was proved by R. Arratia [2] and called by his name.

Depending on the properties of Γ the coalescence of particles can happen. We are interested in the asymptotic properties of clusters and its number in such flows. The key tool in our investigation is a mixing property of the flow with respect to the spatial variable.

Theorem 1 ([3]). *The process $\{x(u,t) - u,\ u \in \mathbb{R}\}$ is a stationary process. Moreover, if $\Gamma(u) \to 0$ as $|u| \to \infty$ then this process has the mixing property.*

Corollary 1. *Consider a Harris flow with a local characteristic Γ such that $\Gamma(u) \to 0$ as $u \to \infty$. Let $\nu_t([u_1,u_2]) = \#\{x([u_1,u_2],t)\}$ be the number of clusters in the Harris flow at the time t with the start points from the interval $[u_1,u_2]$. Assume that $\mathbb{E}\nu_t([0,1]) < \infty$. Then*

$$\lim_{U\to\infty} \frac{\nu_t([0,U])}{U} = \mathbb{E}\nu_t([0,1]) - 1.$$

For the Arratia flow, i.e. for $\Gamma = \mathbb{I}_{\{0\}}$:

$$\lim_{U\to\infty} \frac{\nu_t([0,U])}{U} = \sqrt{\frac{2}{\pi t}}.$$

Theorem 2 ([3]). *Let supp$\Gamma \subset [-c,c]$, $c > 0$. Denote by α the strong mixing coefficient for the process $\{x(u,t) - u,\ u \in \mathbb{R}\}$. Then*

$$\alpha(h) \le 2\sqrt{\frac{2}{\pi}} \int_{h-c}^{\infty} e^{-x^2/2} dx.$$

Using the central limit theorem for stationary sequence proved in [4] one can get asymptotic distribution for a normalized number of clusters in the Arratia flow.

Theorem 3. *Let* $\Gamma = \mathbb{I}_{\{0\}}$.
Then, for every $t > 0$,

$$\frac{\nu_t([0,n]) - \mathbb{E}\nu_t([0,n])}{\sqrt{n}} \Rightarrow N(0, \sigma_t^2), \text{ as } n \to \infty,$$

where $\sigma_t^2 = \frac{3-2\sqrt{2}}{\sqrt{\pi t}}$.

Corollary 2. *Let* $\Gamma = \mathbb{I}_{\{0\}}$. *Then*

$$\sqrt[4]{t}\nu_t([0,1]) - \frac{1}{\sqrt[4]{t}\sqrt{\pi}} \Rightarrow N(0, \sigma_1^2), \ \text{ as } t \to 0.$$

Moreover, we can use the Berry-Esseen inequality for mixing processes proved in [5] to obtain the next theorem:

Theorem 4. *Let* $\Gamma = \mathbb{I}_{\{0\}}$. *Then, for any* $n \geq 1$,

$$\sup_{z \in \mathbb{R}} \left| \mathbb{P}\left\{ \frac{\nu_t([0;n]) - \mathbb{E}\nu_t([0;n])}{\sqrt{n}} \leq z \right\} - \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-r^2/2\sigma_t^2}\, dr \right| \quad \leq \quad Cn^{-1/2} \log n.$$

The assumptions of the law of the iterated logarithm for mixing processes in [6] are also fulfilled and we get:

Theorem 5. *Let* $\Gamma = \mathbb{I}_{\{0\}}$. *Then, for any* $t > 0$,

$$\limsup_{n \to \infty} \frac{\nu_t([0;n]) - \mathbb{E}\nu_t([0;n])}{\sigma_t \sqrt{n \ln \ln n}} = 1 \quad a.s,$$

where $\sigma_t^2 := \dfrac{3 - 2\sqrt{2}}{\sqrt{\pi t}}$.

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MINIMAX INTERPOLATION PROBLEM FOR PERIODICALLY CORRELATED SEQUENCES WITH MISSING OBSERVATIONS

I. I. GOLICHENKO, M. P. MOKLYACHUK

We consider the problem of optimal estimation of the linear functional

$$A_s\zeta=\sum_{l=0}^{s-1}\sum_{j=M_l+1}^{M_l+N_{l+1}}a(j)\zeta(j),\ \ M_l=\sum_{k=0}^{l}(N_k+K_k),\ \ N_0=K_0=0,$$

which depends on the unknown values of a periodically correlated (see [1]) with period T stochastic sequence $\zeta(j)$ from observations of the sequence $\zeta(j)+\theta(j)$ at points $j\in\mathbb{Z}\setminus S$, $S=\bigcup_{l=0}^{s-1}\{M_l+1,\dots,M_l+N_{l+1}\}$, where $\theta(j)$ is an uncorrelated with $\zeta(j)$ periodically correlated stochastic sequence. Assume that the number of missed observations on each of the intervals and the number of observations on each of the intervals are a multiple of T ($K_l=T\cdot K_l^T$ and $N_{l+1}=T\cdot N_{l+1}^T$, $l=0,\dots,s-1$), and coefficients $a(j), j\in S$ are of the form

$$a(j)=a((j-\left[\frac{j}{T}\right]T)+\left[\frac{j}{T}\right]T)=a(\nu+\tilde{j}T)=a(\tilde{j})e^{2\pi i\tilde{j}\nu/T},$$

$$\nu=1,\dots,T,\ \tilde{j}\in\tilde{S},\quad \tilde{S}=\bigcup_{l=0}^{s-1}\left\{M_l^T,\dots,M_l^T+N_{l+1}^T-1\right\},\quad M_l=T\cdot M_l^T,\quad l=0,\dots,s-1.$$

Formulas for the spectral characteristic $\vec{h}(f^{\vec{\zeta}},f^{\vec{\theta}})$ and the mean square error $\Delta(f^{\vec{\zeta}},f^{\vec{\theta}})$ of the optimal estimate of the functional $A_s\zeta$ are obtained in the case where spectral densities of the sequences are exactly known. In this case

$$\vec{h}^{\top}(f^{\vec{\zeta}},f^{\vec{\theta}})=\left(\tilde{A}_s^{\top}(e^{i\lambda})f^{\vec{\zeta}}(\lambda)-\tilde{C}_s^{\top}(e^{i\lambda})\right)\left[f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda)\right]^{-1}=$$

$$=\tilde{A}_s^{\top}(e^{i\lambda})-\left(\tilde{A}_s^{\top}(e^{i\lambda})f^{\vec{\theta}}(\lambda)+\tilde{C}_s^{\top}(e^{i\lambda})\right)\left[f^{\vec{\zeta}}(\lambda)+f^{\vec{\theta}}(\lambda)\right]^{-1},$$

$$\Delta(f^{\vec{\zeta}},f^{\vec{\theta}})=\langle\vec{a}_s^{\zeta},\mathbf{R}_s^{\zeta}\vec{a}_s^{\zeta}\rangle+\langle\vec{c}_s^{\zeta},\mathbf{B}_s^{\zeta}\vec{c}_s^{\zeta}\rangle,$$

where

$$\tilde{A}_s(e^{i\lambda})=\sum_{\tilde{j}\in\tilde{S}}\vec{a}(\tilde{j})e^{i\tilde{j}\lambda},$$

$$\vec{a}(\tilde{j})=(a_1(\tilde{j}),\dots,a_T(\tilde{j}))^{\top},\ a_\nu(\tilde{j})=a(\nu+\tilde{j}T),\ \nu=1,\dots,T,$$

$$\vec{a}_s^{\zeta}=\left(\vec{a}^{\top}(0),\dots,\vec{a}^{\top}(N_1^T-1),\dots,\vec{a}^{\top}(M_{s-1}^T),\dots,\vec{a}^{\top}(M_{s-1}^T+N_s^T-1)\right)^{\top},$$

$$\tilde{C}_s(e^{i\lambda})=\sum_{\tilde{j}\in\tilde{S}}\vec{c}(\tilde{j})e^{i\tilde{j}\lambda},$$

$$\vec{c}_s^{\zeta}=\left(\vec{c}^{\top}(0),\dots,\vec{c}^{\top}(N_1^T-1),\dots,\vec{c}^{\top}(M_{s-1}^T),\dots,\vec{c}^{\top}(M_{s-1}^T+N_s^T-1)\right)^{\top}=(\mathbf{B}_s^{\zeta})^{-1}\mathbf{D}_s^{\zeta}\vec{a}_s^{\zeta}.$$

Formulas that determine the least favorable spectral densities and the minimax spectral characteristics are proposed for some classes of admissible spectral densities.

Formulation and investigation of problems of extrapolation, interpolation and filtering of linear functionals which depend on the unknown values of periodically correlated sequences and processes without missing observations are presented in [2].

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FIRST EXIT TIMES FOR EXPONENTIALLY LIGHT JUMP DIFFUSIONS – A LARGE DEVIATIONS APPROACH

ANDRÉ DE OLIVEIRA GOMES, MICHAEL HÖGELE

It is a well known fact that, under certain conditions, the solution trajectories of a dynamical system given by a differential equation written in a gradient form never leave the domain of attraction of its stable states. The perturbation of such systems, in low intensity, by a Brownian Motion, is a very well developed field of study and it is known as *Freidlin-Wentzell theory*. Informally, with Gaussian perturbations with small intensity, it is possible that the trajectories of the stochastic perturbed equations leave the domain of attraction of the stable state and such exit happens to occur with small probabilities but exponentially large in the intensity parameter that tunes the noise. Outside the realm of Gaussian perturbations, in the vanishing noise regime, other studies were conducted and it was observed different regimes of deviations, polynomially large in the noise parameter. We present a certain class of perturbations by Lévy noises in such a way that it is possible to characterize the exit rates of the domains of attraction using large deviations principles. Our stochastic processes are jump processes that have an exponentially light integrability property in the tails and we characterize the problem of the first exit time in terms of the parameter of lightness of such tails observing a phase transition. When the jump processes are super-exponentially light the first exit time is studied in a large deviations regime and when the jump measure is sub-exponentially light such study is conducted by means of moderate deviations principles. This talk is based on joint work with Michael Högele (U. de los Andes, Bogotá Colombia).

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NESTED OCCUPANCY SCHEMES IN RANDOM ENVIRONMENTS

ALEXANDER IKSANOV

Let $(P_r)_{r\in\mathbb{N}}$ be a collection of positive random variables satisfying $\sum_{r\geq 1} P_r = 1$ a.s. Assume that, given $(P_r)_{r\in\mathbb{N}}$, 'balls' are allocated independently over an infinite collection of 'boxes' 1, 2, ... with probability P_r of hitting box r, $r \in \mathbb{N}$. The occupancy scheme arising in this way is called the *infinite occupancy scheme in the random environment* $(P_r)_{r\geq 1}$.

A popular model of the infinite occupancy scheme in the random environment assumes that the probabilities $(P_r)_{r\in\mathbb{N}}$ are formed by an enumeration of the a.s. positive points of

$$\{e^{-X(t-)}(1 - e^{-\Delta X(t)}) : t \geq 0\}, \tag{1}$$

where $X := (X(t))_{t\geq 0}$ is a subordinator (a nondecreasing Lévy process) with $X(0) = 0$, zero drift, no killing and a nonzero Lévy measure, and $\Delta X(t)$ is a jump of X at time t. Since the closed range of the process X is a regenerative subset of $[0, \infty)$ of zero Lebesgue measure, one has $\sum_{r\geq 1} P_r = 1$ a.s. When X is a compound Poisson process, collection (1) transforms into a *residual allocation model*

$$P_r := W_1 W_2 \cdot \ldots \cdot W_{r-1}(1 - W_r), \quad r \in \mathbb{N}, \tag{2}$$

where W_1, $W_2, \ldots$ are i.i.d. random variables taking values in $(0, 1)$.

Next, following [1] I define a nested infinite sequence of the infinite occupancy schemes in random environments. This means that I construct a nested sequence of environments (random probabilities) and the corresponding 'boxes' so that the same collection of 'balls' is thrown into all 'boxes'. To this end, I use a weighted branching process with positive weights which is nothing else but a multiplicative counterpart of a branching random walk.

The nested sequence of environments is formed by the weights $(R(u))_{|u|=1} = (P_r)_{r\in\mathbb{N}}$, $(R(u))_{|u|=2}, \ldots$, say, of the subsequent generations individuals in a weighted branching process. Further, I identify individuals with 'boxes'. At time $j = 0$, all 'balls' are collected in the box $\oslash$ which corresponds to the initial ancestor. At time $j = 1$, given $(R(u))_{|u|=1}$, 'balls' are allocated independently with probability $R(u)$ of hitting box u, $|u| = 1$. At time $j = k$, given $(R(u))_{|u|=1}, \ldots, (R(u))_{|u|=k}$, a ball located in the box u with $|u| = k - 1$ is placed independently of the others into the box ur, $r \in \mathbb{N}$ with probability $R(ur)/R(u)$.

Assume that there are n balls. For $r = 1, 2, \ldots, n$ and $j \in \mathbb{N}$, denote by $K_{n,j,r}$ the number of boxes in the jth generation which contain exactly r balls and set

$$K_{n,j}(s) := \sum_{r=\lceil n^{1-s}\rceil}^{n} K_{n,j,r}, \quad s \in [0, 1],$$

where $x \mapsto \lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$ is the ceiling function. With probability one the random function $s \mapsto K_{n,j}(s)$ is right-continuous on $[0, 1)$ and has finite limits from the left on $(0, 1]$ and as such belongs to the Skorokhod space $D[0, 1]$. I am going to present sufficient conditions which ensure functional weak convergence of $(K_{n,j_1}(s), \ldots, K_{n,j_m}(s))$, properly normalized and centered, for any finite collection of indices $1 \leq j_1 < \ldots < j_m$ as the number n of balls tends to ∞. If time permits, I shall discuss specializations of the general result to $(P_r)_{r\in\mathbb{N}}$ given by (1) and (2).

The talk is based on a work in progress, joint with Sasha Gnedin (London).

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LIMIT THEORY FOR MULTI-DIMENSIONAL RENEWAL SETS

ANDRII ILIENKO

Consider a sequence $(\xi_i, i \in \mathbb{N})$ of i.i.d. random variables with a finite mean $\mu > 0$, their partial sums $S_n = \sum_{i \le n} \xi_i$, and the corresponding renewal process $N = \big(N(t), t > 0\big)$ given by $N(t) = \min\{n \colon S_n \ge t\}$. The asymptotic properties of N are well-known (see, e.g., [1] for strong LLNs, LILs, and distributional limit theorems for renewal processes).

In the multi-dimensional setting, i.e. for a multi-indexed family $\big(\xi_i, i \in \mathbb{N}^d\big)$ and respective partial sums S_n over boxes $\{1, \dots, n_1\} \times \dots \times \{1, \dots, n_d\}$, the above definition of the renewal process is no more applicable due to the lack of a natural total order in $\mathbb{N}^d$. Consider the renewal lattice set $\mathcal{M}_t = \big\{n \in \mathbb{N}^d \colon S_n \ge t\big\}$: the family $(\mathcal{M}_t, t > 0)$ may be regarded as a set-valued stochastic process, which is a multi-dimensional counterpart of the one-dimensional renewal process N.

The talk provides a short overview of some known limit theorems concerning the asymptotic behaviour of multi-dimensional renewal processes as well as new asymptotic results about the location and the shape of renewal sets. In more detail, we show that appropriately rescaled and smoothed renewal sets converge to the (non-random) set $\mathcal{H} = \big\{x \in \mathbb{R}_+^d \colon x_1 \cdot \ldots \cdot x_d \ge \mu^{-1}\big\}$. The rate of convergence (in the form of the Marcinkiewicz-Zygmund strong law of large numbers), the law of the iterated logarithm, and the weak invariance principle are studied as well. The strong LLN and the LIL are expressed in the form of set inclusions and in terms of distances between sets, the latter being the Hausdorff metric as well as the symmetric difference one.

More details on results presented in the talk can be found in [2].

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ON GROWTH RATE OF SUMS FOR M-REGRESSION SEQUENCES

MARYNA ILIENKO

This talk continues the main line of investigation carried out in [2] where some results on the rate of growth for sums of elements of the first order regression sequences were obtained. The technique used in [2] is based on methods (see [1]) which allow to extend the setting of the problem to the case of m-regression. Thus, consider the m-regression sequence of random variables (ξ_k):

$$\xi_k = b_k^{(1)}\xi_{k-1} + b_k^{(2)}\xi_{k-2} + ... + b_k^{(m)}\xi_{k-m} + \beta_k\theta_k, \quad k \geq 1, \tag{1}$$
$$\xi_{1-m} = ... = \xi_{-1} = \xi_0 = 0,$$

where (β_k) is a nonrandom real sequence, $(b_k^{(j)}; \quad 1 \leq j \leq m, \quad k \geq 1)$ is a nonrandom set of reals, and (θ_k) is a sequence of independent symmetric random variables such that $\mathbb{P}\{\theta_k = 0\} < 1$, $k \geq 1$. Let

$$S_n = \sum_{k=1}^{n} \xi_k, \quad n \geq 1,$$

and consider the random series

$$\sum_{n=1}^{\infty} \frac{S_n}{n^{1+1/p}} \tag{2}$$

for $p > 0$. The domain of convergence for the above series in p may be regarded as a form of specifying of the growth rate for S_n. So, we are interested in necessary and sufficient conditions for the convergence almost surely of the series (2). Although general necessary and sufficient conditions in terms of coefficients of the sequence (1) are bulky, they appear to be quite simple at least in some specific cases.

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CONSISTENCY OF THE KOENKER-BASSETT ESTIMATOR IN LINEAR REGRESSION MODEL

A. V. IVANOV, N. V. KAPTUR, I. N. SAVYCH

Consider a regression model

$$X_j = g(j,\theta) + \varepsilon_j,$$

$j = \overline{1,N}, N \in \mathbb{N}$, where $g(j,\theta) = \sum\limits_{i=1}^{q} \theta_i g_i(j)$, $j = \overline{1,N}$, $\theta = (\theta_1, ..., \theta_q) \in \Theta^c$, $\Theta \subset \mathbb{R}^q$, is an open bounded set.

Regarding ε_j suppose

A1. ε_j, $j \in \mathbb{Z}$, is a local functional of Gaussian time series, $\varepsilon_j = G(\xi_j)$, $G(x)$, $x \in \mathbb{R}$, is nonrandom Borel function, and what's more

$$\mathsf{E}\,\varepsilon_0 = 0,\ \mathsf{E}\,\varepsilon_0^2 < \infty.$$

A2. ξ_j, $j \in \mathbb{Z}$, is a Gaussian stationary time series with zero mean and covariance function

$$B(j) = \mathsf{E}\,\xi_j\xi_0 = \sum_{l=0}^{r} A_l B_{\alpha_l,\chi_l}(j), j \in \mathbb{Z}, r > 0,$$

where $A_l > 0$, $\sum\limits_{l=0}^{r} A_l = 1$, $B_{\alpha_l,\chi_l}(j) = \frac{\cos(\chi_l j)}{(1+j^2)^{\frac{\alpha_l}{2}}}$, $l = \overline{0,r}$, $0 = \chi_0 < \chi_1 < ... < \chi_r < \pi$, $\alpha_l \in (0,1)$.

Let $F(x)$ be a distribution function of ε_0.

A3. $F(0) = \beta$, $\beta \in (0,1)$.

Introduce a loss function

$$\rho_\beta(x) = \begin{cases} \beta x, & x \geq 0, \\ (\beta - 1)x, & x < 0. \end{cases}, \quad \beta \in (0,1).$$

Definition. Koenker-Bassett estimator of parameter $\theta \in \Theta$ is said to be any random vector $\hat{\theta}_N = \hat{\theta}_N \left(X_j,\ j \in \overline{1,N}\right) \in \Theta^c$ for which

$$Q_N(\hat{\theta}_N) = \min_{\tau \in \Theta^c} Q_N(\tau),\ \ Q_N(\tau) = \sum_{j=1}^{N} \rho_\beta \left(X_j - g(j,\tau)\right).$$

Set $d_N^2 = diag\,(d_{iN}^2)_{i=1}^q$, $\quad d_{iN}^2 = \sum\limits_{j=1}^{N} g_i^2(j)$, and assume

$$0 < \varliminf_{N\to\infty} N^{-1/2} d_{iN} \leq \varlimsup_{N\to\infty} N^{-1/2} d_{iN} < \infty, \quad i = \overline{1,q}.$$

Perform the change of variables in the regression function $u = N^{-1/2} d_N (\tau - \theta)$ and set

$$h(j,u) = g(j, \theta + N^{1/2} d_N^{-1} u),$$

supposing θ is a true parameter value.

The parametric set Θ transforms into $\tilde{U}_N(\theta) = N^{-1/2} d_N(\Theta - \theta)$. The sense of this change of variables is that the Koenker-Bassett estimator $\hat{\theta}_N$ transforms into the normed estimator $\bar{u}_N = N^{-1/2} d_N(\hat{\theta}_N - \theta)$.

Let the following conditions be fulfilled.

B. (i) For any $\varepsilon > 0$ and $r > 0$ there exist $\delta = \delta(r, \varepsilon) > 0$ such that

$$\sup_{\substack{u_1, u_2 \in V^c(r) \bigcap \tilde{U}_N^c(\theta), \\ \|u_1 - u_2\| \le \delta}} N^{-1} \sum_{j=1}^{N} |h(j, u_1) - h(j, u_2)| \le \varepsilon,$$

where $V^c(r) = \{u \in \mathbb{R}^q : \|u\| \le r\}$.

(ii) For any $r > 0$ there exists $\sigma = \sigma(r) < \infty$ such that

$$\sup_{u \in V^c(r) \bigcap \tilde{U}_N^c(\theta)} N^{-1} \sum_{j=1}^{N} (h(j, u) - h(j, 0))^2 \le \sigma.$$

C. For any $r > 0$ there exist $\Delta(r) > 0$ such that

$$\inf_{u \in \tilde{U}_N^c(\theta) \setminus V^c(r)} N^{-1} \mathsf{E}\, Q_N(\theta + N^{\frac{1}{2}} d_N^{-1} u) \ge \mathsf{E}\, \rho_\beta(\varepsilon_0) + \Delta(r),$$

Theorem. *Under assumptions* **A1-A3, B, C** *the normed Koenker-Bassett estimator is weakly consistent, namely: for any $r > 0$*

$$\mathsf{P}(\|\bar{u}_N\| \ge r) = O\left(B\left(N\right)\right) \quad as \quad N \to \infty.$$

The proof of this theorem is based on the proof of a similar theorem for a nonlinear continuous-time regression model, see [1].

Let us give sufficient conditions for validity of **C.** Random variable ε_0 has absolutely continuous distribution function $F(x)$, and next relations hold

$$\sup_{j \in N} \sup_{\tau_1, \tau_2 \in \Theta^c} |g(j, \tau_1) - g(j, \tau_2)| = g_0 < \infty, \qquad \inf_{|x| \le g_0} p(x) = p_0 > 0,$$

where $p(x) = F'(x)$. It can be proved, that $\mathsf{E}\, \rho_\beta(\varepsilon_j \pm g) - \mathsf{E}\, \rho_\beta(\varepsilon_0) \ge \frac{1}{2} p_0 g^2$ for $g \in [0, g_0]$.

Thus, $N^{-1} \mathsf{E}\, Q_N(\theta + N^{\frac{1}{2}} d_N^{-1} u) \ge \mathsf{E}\, \rho_\beta(\varepsilon_0) + \frac{1}{2} p_0 N^{-1} \sum_{j=1}^{N} (h(j, u) - h(j, 0))^2$,

and the validity of the condition **C** depends on the ability of the regression function to distinguish the parameters.

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ASYMPTOTIC NORMALITY OF THE LEAST SQUARES ESTIMATOR OF THE TEXTURED SURFACE SINUSOIDAL MODEL PARAMETERS

A. V. IVANOV, O. V. MALYAR

Consider the observation model

$$X(t_1,t_2) = g(t_1,t_2;\theta^0) + \varepsilon(t_1,t_2), \quad t=(t_1,t_2)\in\mathbb{R}_+^2,$$
$$g(t_1,t_2;\theta^0) = \sum_{k=1}^{N}(A_k^0\cos(\lambda_k^0 t_1 + \mu_k^0 t_2) + B_k^0 \sin(\lambda_k^0 t_1 + \mu_k^0 t_2)),$$
$$\theta^0 = (A_1^0, B_1^0, \lambda_1^0, \mu_1^0, ..., A_N^0, B_N^0, \lambda_N^0, \mu_N^0),$$

$(A_k^0)^2 + (B_k^0)^2 > 0$, $k=\overline{1,N}$, θ^0 is the vector of true values of unknown parameters; $\varepsilon = \{\varepsilon(t_1,t_2), (t_1,t_2)\in\mathbb{R}^2\}$ is the random noise defined on a probability space $(\Omega,\Im,\mathbb{P})$. We state the following assumptions.

N. ε is a mean square continuous and almost surely continuous homogeneous Gaussian field with zero mean and covariance function $B(t_1,t_2) = \mathsf{E}\varepsilon(t_1,t_2)\times\varepsilon(0,0)$, $(t_1,t_2)\in\mathbb{R}^2$, which satisfies one of the following conditions:

(**i**) ε is isotropic field and $B(t_1,t_2) = B(\|t\|) = L(\|t\|)\|t\|^{-\alpha}$, $\alpha\in(0,1)$, where L is a monotonically non-derceasing slowly varying at infinity function, $t=(t_1,t_2)$, $\|t\| = (t_1^2+t_2^2)^{1/2}$;

(**ii**) $\int_{\mathbb{R}^2} |B(t_1,t_2)|dt_1dt_2 < \infty$.

R1. $(\lambda_k^0,\mu_k^0) < (\lambda_{k+1}^0,\mu_{k+1}^0)$, $k=\overline{1,N-1}$, and all the values λ_j^0, μ_j^0, $i,j=\overline{1,N}$, are positive and different.

Consider two families of monotonically non-decreasing open sets

$$\Lambda_T\subset\Lambda(\underline{\lambda},\overline{\lambda}),\ M_T\subset M(\underline{\mu},\overline{\mu}),\ T\geq T_0>0,$$

containig true values of parameters λ^0, μ^0, and satisfying the conditions

R2.
$$\lim_{T\to\infty}\inf_{\substack{1\le j\le N-1\\ \lambda\in\Lambda_T}} T(\lambda_{j+1}-\lambda_j) = \lim_{T\to\infty}\inf_{\substack{1\le j\le N-1\\ \mu\in M_T}} T(\mu_{j+1}-\mu_j) = \infty,$$
$$\lim_{T\to\infty}\inf_{\lambda\in\Lambda_T} T\lambda_1 = \lim_{T\to\infty}\inf_{\mu\in M_T} T\mu_1 = \infty.$$

Definition. The least squares estimator (LSE) in the Walker sence of the vector parameter θ^0 obtained by the observation of the field $X(t_1,t_2), (t_1,t_2)\in[0,T]\times[0,T]$, is called any random vector $\theta_T = (A_{1T}, B_{1T}, \lambda_{1T},\ \mu_{1T},\ldots,A_{NT},B_{NT},\lambda_{NT},\mu_{NT})$, such that it is a point of absolute minimum of

$$Q_T(\theta) = T^{-2}\int_0^T\int_0^T [X(t_1,t_2)-g(t_1,t_2;\theta)]^2dt_1dt_2$$

on the parametric set $\Theta \subset \mathbb{R}^{4N}$ where amplitudes $A_k,\ B_k,\ k = \overline{1, N}$, can take any values and angular frequencies λ, μ take values in the sets $\Lambda_T^c,\ M_T^c$.

If the assumption $\mathbf{N(ii)}$ is fulfilled, then continuous bounded spectral density $f(\lambda_1, \lambda_2)$, $(\lambda_1, \lambda_2) \in \mathbb{R}^2$, of the field ε exists. In case of fulfillment of the condition $\mathbf{N(i)}$ an additional assumption must be introduced.

NS. The random noise $\varepsilon(t),\ t \in \mathbb{R}^2$, has a spectral density function

$$f(\lambda_1, \lambda_2) = f(\|\lambda\|) = c(\alpha)\|\lambda\|^{\alpha-2} L_s\Big(\frac{1}{\|\lambda\|}\Big)$$

where $c(\alpha) = \dfrac{\Gamma(\frac{2-\alpha}{2})}{2^\alpha \pi \Gamma(\frac{\alpha}{2})}$, $\alpha \in (0,1)$ and coinsides with α from the condition $\mathbf{N(i)}$, L_s is a locally bounded slowly varying at infinity function such that f is continuous on the set $\mathbb{R}^2 \setminus \{0\}$ and $f(\|\lambda\|) \uparrow \infty$, as $\|\lambda\| \to 0$.

Theorem. *If the assumptions* $\mathbf{R1}$, $\mathbf{R2}$, $\mathbf{N(ii)}$ *or* $\mathbf{N(i)}$ *and* $\mathbf{NS}$ *are satisfied, then the normed LSE in the Walker sence* $(T(A_{1T} - A_1^0),\ T(B_{1T} - B_1^0),\ T^2(\lambda_{1T} - \lambda_1^0),\ T^2(\mu_{1T} - \mu_1^0), \ldots, T(A_{NT} - A_N^0), T(B_{NT} - B_N^0), T^2(\lambda_{NT} - \lambda_N^0), T^2(\mu_{NT} - \mu_N^0))$ *is asymptotically normal with zero mean and covariance matrix* $\Psi(\theta^0)$ *where* $\Psi(\theta^0)$ *is a block diagonal matrix with the blocks*

$$\Psi_k = \frac{8\pi^2 f(\lambda_k^0, \mu_k^0)}{(A_k^0)^2 + (B_k^0)^2} \begin{bmatrix} (A_k^0)^2 + 7(B_k^0)^2 & -6A_k^0 B_k^0 & -6B_k^0 & -6B_k^0 \\ -6A_k^0 B_k^0 & 7(A_k^0)^2 + (B_k^0)^2 & 6A_k^0 & 6A_k^0 \\ -6B_k^0 & 6A_k^0 & 12 & 0 \\ -6B_k^0 & 6A_k^0 & 0 & 12 \end{bmatrix},$$

$$k = \overline{1, N}.$$

Let us provide a sketch of the proof. We consider a general nonlinear regression model and prove a linearization theorem for the LSE of its parameters. Further we prove asymptotic uniqueness in probability for this LSE and apply Brouwer fixed point theorem to reduce the proof of the Theorem to calculating of the LSE asymptotic covariance matrix in the linearized model. We check also that our trigonometric model satisfies the conditions of these general theorems.

Consistency of the LSE in the Walker sence of the parameter θ^0 is proved in [1].

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ON THE WHITTLE ESTIMATOR FOR SPECTRAL DENSITY PARAMETERS OF LINEAR RANDOM NOISE IN NONLINEAR REGRESSION MODEL

A. V. IVANOV, I. V. ORLOVSKYI

Consider a regression model

$$X(t) = g(t,\, \alpha_0) + \varepsilon(t),\ t \geq 0,$$

where $g : (-\Delta; \infty) \times \mathcal{A}_\gamma \to \mathbb{R}$ is a continuous function, true parameter value α_0 belongs to an open bounded convex set $\mathcal{A} \subset \mathbb{R}^q$, $\mathcal{A}_\gamma = \bigcup\limits_{\|e\| \leq 1} (\mathcal{A} + \gamma e)$, γ, Δ are some positive numbers, and random noise $\varepsilon = \{\varepsilon(t),\ t \in \mathbb{R}\}$ is measurable stationary linear process in the sense that it can be represented in the form

$$\varepsilon(t) = \int\limits_{\mathbb{R}} a(t-s) d\zeta(s),\ t \in \mathbb{R}, \tag{1}$$

where $a(t),\ t \in \mathbb{R}$, is a non-random function, $a \in L_2(\mathbb{R})$, $\zeta(A),\ A \in \mathfrak{B}$, is a homogeneous random measure with finite seconds moments and independent values over disjoint sets. Various conditions which ensure that (1) is well-defined can be found, for example, in [1].

Let all the moments of ε exist, $E\varepsilon(t) = 0$. Suppose random process ε has all spectral densities of higher orders (see, for example [2]) that can be written explicitly as

$$f_r(\lambda_1,\, \ldots,\, \lambda_{k-1}) = d_r \cdot \widehat{a}(-\lambda_1 - \ldots - \lambda_{r-1}) \cdot \prod_{j=1}^{r-1} \widehat{a}(\lambda_j),$$

where $\widehat{a}(\lambda) = \dfrac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} a(t) e^{-i\lambda t} dt$ is the Fourier transform of a, d_r is r-th order cumulant of random variable $\zeta\big([0;\, 1]\big)$.

Assume that $\widehat{a}(\lambda) = \widehat{a}(\lambda,\, \theta_{a0})$, $d_k = d_k(\theta_{d0})$. Then ordinary spectral density of random process ε is of the form

$$f_2(\lambda) = d_2 \widehat{a}(\lambda) \widehat{a}(-\lambda) = d_2 |\widehat{a}(\lambda)|^2 = f(\lambda,\, \theta_0),\ \theta_0 = (\theta_{a0},\, \theta_{d0}) \in \Theta,$$

where $\Theta \in \mathbb{R}^m$ is an open bounded convex set, $f(\lambda,\, \theta) > 0$ is defined on the set $\mathbb{R} \times \Theta_\tau$, $\Theta_\tau = \bigcup\limits_{\|e\| \leq 1} (\Theta + \tau e),\ \tau > 0$.

Estimation of unknown parameter θ_0 of random noise spectral density is considered in the talk. Regression function becomes interfering signal from that point of view. So we firstly need to neutralize the influence of $g(t,\, \alpha_0)$ and only after that construct estimator of spectral density parameter. It means that we have to start from the estimation of regression function parameter α_0. The least squares estimator was chosen for this purpose as its definition does not need any information about random noise characteristics.

Definition 1. The least squares estimator of the unknown parameter $\alpha \in \mathcal{A}$ obtained by the observations $\{X(t),\ t \in [0,\, T]\}$ is said to be any random vector $\widehat{\alpha}_T = (\widehat{\alpha}_{1T},\, \dots,\, \widehat{\alpha}_{qT}) \in \mathcal{A}^c$, $\mathcal{A}^c$ is a closure of $\mathcal{A}$ having the property

$$Q_T\left(\widehat{\alpha}_T\right) = \min_{\alpha \in \mathcal{A}^c} Q_T(\alpha),\ Q_T(\alpha) = \int\limits_0^T \left(X(t) - g(t,\, \alpha)\right)^2 dt.$$

The Whittle estimator was chosen for the estimation of parameter θ_0 of spectral density $f(\lambda, \theta)$ as it plays an important role in parameter estimation in the frequency domain and it is one of the most popular estimator in applications.

Let us introduce the residual periodogram

$$I_T(\lambda,\, \widehat{\alpha}_T) = \frac{1}{2\pi T}\left|\int\limits_0^T \left(X(t) - g(t,\, \widehat{\alpha}_T)\right) e^{-it\lambda} dt\right|^2,\ \lambda \in \mathbb{R},$$

and consider the Whittle contrast field

$$U_T(\theta,\, \widehat{\alpha}_T) = \int\limits_{-\infty}^{\infty} \left(\ln f(\lambda,\, \theta) + \frac{I_T\left(\lambda,\, \widehat{\alpha}_T\right)}{f(\lambda,\, \theta)}\right) \omega(\lambda) d\lambda,\ \theta \in \Theta^c,$$

where $\omega(\lambda)$, $\lambda \in \mathbb{R}$, is some even positive bounded and Lebesgue measurable weight function.

Definition 2. The minimum contrast estimator of the unknown parameter $\theta_0 \in \Theta$ is said to be any random vector $\widehat{\theta}_T = \left(\widehat{\theta}_{1T}, ..., \widehat{\theta}_{mT}\right)$ having the property

$$U_T\left(\widehat{\theta}_T,\, \widehat{\alpha}_T\right) = \min_{\theta \in \Theta^c} U_T\left(\theta, \widehat{\alpha}_T\right).$$

Asymptotic properties of the Whittle estimator of the spectral density parameter of stationary Gaussian random noise in the nonlinear regression models was considered in [3].

Sufficient conditions of consistency and asymptotic normality of the Whittle estimator of spectral density parameter of linear stationary process in nonlinear regression models are presented in the talk. Obtained statements continue research of [3] and extend them on class of linear random noise that now is not necessary Gaussian.

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INVARIANT PROBLEM AND ITS TRACTABILITY FOR STOCHASTIC SYSTEMS

ELENA KARACHANSKAYA

A mathematical formulation of open systems described by stochastic differential equations is actual and thus an invariant problem for stochastic systems is very important.

In connection with this problem it is of interest to discuss available approaches for an invariant definition.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_0^T, P)$ be a probability space with filtration. Suppose that $w(t) = (w_1(t), ..., w_m(t))^T$ is an $m-$dimensional Wiener process. Consider a random process $x(t) : [0,T] \to \mathbb{R}^n$, which is a solution to a system of Itô SDE

$$\begin{gathered} dx_i(t) = a_i(t, x(t))dt + \sum_{k=1}^{m} b_{ik}(t, x(t))dw_k(t), \\ x(t; x(0))\big|_{t=0} = x(0), \end{gathered} \tag{1}$$

whose coefficients (in general, random functions) satisfy the conditions of the existence and uniqueness of a solution and the following smoothness conditions:

$$a_i(t; \mathbf{x}) \in \mathcal{C}^{1,1}_{t,x}, \quad b_{ij}(t; \mathbf{x}) \in \mathcal{C}^{1,2}_{t,x}. \tag{2}$$

Let us consider a function $(t, x) \to \varphi(t, x)$: $[0, T] \times \mathbb{R}^n \to \mathbb{R}^1$ under condition $\varphi(t, x) \in \mathcal{C}^{1,2}_{t,x}$. A nonrandom function $\varphi(t, x)$ is called a first integral of the system (1) if it preserves a constant value with probability 1 for every realization of a random process $x(t)$ that is a solution to this system:

$$\varphi(t, x(t, x(0)) = \varphi(0, x(0)) \quad (P - a.s.).$$

A first integral concept for Itô diffusion systems and a stochastic first integral concept for that systems with jumps were introduced by V. Doobko [1, 2, 3]. Criterions for determing whether the same function is a first integral for the given SDE system were obtained in [1, 4].

Remark 1. Both a direct first integral and a back first integral for diffusion system was introduced by N. Krylov and B. Rozovskiy [5]. Conditions for ones were not established.

Theorem 1. [1] *Let $x(t)$ be a solution to the system of Itô SDE* (1) *with conditions* (2). *A norandom function $\varphi(t, x) \in \mathcal{C}^{1,2}_{t,x}$ is a first integral of system* (1) *if and only if it satisfies the conditions:*

(1) $\dfrac{\partial \varphi(t; x)}{\partial t} + \dfrac{\partial \varphi(t; x)}{\partial x_i}\Big[a_i(t; x) - \dfrac{1}{2}\, b_{j\,k}(t; x) \dfrac{\partial b_{i\,k}(t; \mathbf{x})}{\partial x_j}\Big] = 0;$

(2) $b_{i\,k}(t; x) \dfrac{\partial \varphi(t; x)}{\partial x_i} = 0,$ *for all* $k \in \{1, \ldots, m\}$.

In [6] was represented new conditions for invariant function for diffusion systems. But really, these conditions are special case of the conditions above [4].

Theorem 2. *Let a function $\varphi(t;x)$ is a first integral for* (1) *under conditions* (2)*. Then for any integrated function $\mu(t)$ a function $u(t,x) = \varphi(t,x) - \int_0^t \mu(\tau)$ is a first integral for system* (1)*.*

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I. I. GIKHMAN WORKS ON MULTIPARAMETER MARTINGALES AND RELATED TOPICS

O.I. KLESOV

I. I. Gikhman contribution to probability and stochastic processes is acknowledged, in the first place, by his development of the theory of stochastic differential equations. However a considerable part of Gikhman's scientific activity was directed toward the development of the notion and properties of *multiparameter martingales*. The aim of this talk is to briefly survey some of I. I. Gikhman's contributions to the topic of multiparameter martingales.

The first Gikhman paper on this topic is [2] where, although, he mentions an earlier publication [1]. Gikhman studies on multiparameter martingales are done independently of those by Cairoli [20] or Wong and Zakai [21] whom usually the priority is given to. As far as the question on who is the first to study multiparameter martingales is concerned, my knowledge of the literature is that K. Krickeberg [19] is that person. However Krickeberg himself in *My Encounters with Martingales*, J. Contemporary Math. Anal., **44** (2009), pp. 9–13, mentioned that J. Dieudonné paper [18] is a source for the paper [19].

I. I. Gikhman's definition of a multiparameter martingale [2] is slightly different of the definitions in [20] or in [21]. One of the main differences is that I. I. Gikhman avoids the so-called condition A_4 and, nevertheless, obtains several important results for multiparameter martingales.

Gikhman starts the paper [7] with the sentence "*The passage from the theory of martingales of one argument to that of martingales of two arguments is beset by a number of difficulties of principle, whereas the passage from two to several arguments is mainly a problem of the complexity of the notation and formulations*". This explains the Gikhman concern on the two-parameter case which, he believed, does not restrict the generality. One of the main difficulties mentioned by Gikhman "... *consists in the fact that the technique of stopping moments is not suitable in the two-argument case*".

After the paper [7], I. I. Gikhman enlarged his interest to the topic and studied more general random fields and solutions of stochastic partial differential equations with the help of multi-parameter martingales.

Early I. I. Gikhman publications concerning the multi-parameter martingales are [2]–[7]. Results on extensions to random fields are [8]–[15]. Related papers published by Gikhman's former students are [16]–[17]. Some pioneering papers on multiparameter martingales are [18]–[22].

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ALMOST SURE ASYMPTOTIC PROPERTIES OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATION WITH SEPARABLE VARIABLES

O. I. KLESOV, O. A. TYMOSHENKO

Stochastic differential equations are one of the effective models of stochastic processes that are used in many fields of science such as insurance and financial mathematics, economics, control theory and many others (see Øksendal [6]).

We study the behior of a solution of the following non-homogeneous stochastic differential equation with separation of stochastic and deterministic variables

$$\begin{aligned} &dX(t) = \varphi(t)g(X(t))dt + \theta(t)\sigma(X(t))dw(t), \qquad t \geq 0; \\ &X(0) = b > 0. \end{aligned} \tag{1}$$

Here φ, g, θ, and σ are some deterministic functions.

The asymptotic behavior of solutions of such equations are esxpressedd in terms of the related ordinary differential equation

$$\begin{aligned} &dx(t) = \varphi(t)g(x(t))dt, \qquad t \geq 0; \\ &x(0) = b > 0. \end{aligned}$$

The case of a homogeneous equation (i.e. if $\varphi(t) \equiv 1$) is considered by Gihman and Skorohod [3] and Keller et. al [4]. A partial case of equation (1) is investigated by Appleby et al. [1]. More general cases are studied in [2, 8].

The following function

$$\Phi(t) = \int_0^t \varphi(s)ds, \qquad t \geq 0,$$

is involved in the statement of the result below. We assume that $\lim_{t\to\infty} \Phi(t) = \infty$.

A crucial role in our approach is played by the assumption

$$\lim_{t\to\infty} X(t) = \infty \qquad \text{almost surly.} \tag{2}$$

Some general conditions for (2) are given by Klesov and Tymoshenko [5].

Population Growth Model. Consider the Cauchy problem

$$dX(t) = \varphi(t)X(t)dt + \beta X(t)dw(t), \;\; t \geq 0; \;\; X(0) = 1. \tag{3}$$

A solution of problem (3) describes the growth of a population with unit initial size (see [6]), where $X(\cdot)$ is the size of population at time t; $\varphi(\cdot)$ is relative growth rate of the population that depends on time; $w(\cdot)$ is a Wiener process; $\beta \in (0, +\infty)$. Let $\varphi(\cdot)$ be a positive continuous function.

Theorem 1. *Let $X(\cdot)$ be a solution of problem* (3)*. Assume that*

$$K = \lim_{t\to\infty} \frac{\Phi(t)}{t} > \frac{1}{2}\beta^2,$$

Then

$$\lim_{t\to\infty}\frac{\ln X(t)}{\Phi(t)} = 1-\frac{\beta^2}{2K}\quad a.s.$$

Rendleman–Bartter Model. Consider the Cauchy problem for the Rendleman–Bartter Model (see [7])

$$dX(t)=\varphi(t)X(t)dt+\theta(t)X(t)dw(t),\ t\ge 0;\ X(0)=b>0. \tag{4}$$

Here $\varphi(\cdot)$ represents an expected instantaneous rate of change in the interest rate, $\theta(\cdot)$ is a volatility parameter, and $w(\cdot)$ is a Wiener process.

Theorem 2. *Let $\varphi(\cdot)$ and $\theta(\cdot)$ be continuous functions and let $X(\cdot)$ be a solution of Cauchy problem* (4)*. Assume that*

$$\lim_{t\to\infty}\frac{1}{\Phi(t)}\int\limits_0^t \theta^2(s)ds = L,\ L\in[0;\infty);\quad and\quad \sum_{k=0}^{\infty}\frac{\Phi\left(2^{k+1}\right)}{\Phi^2\left(2^k\right)}<\infty.$$

Then

$$\lim_{t\to\infty}\frac{\ln X(t)}{\Phi(t)} = 1-\frac{1}{2}L\quad a.s.$$

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ON SOME INTEGRO-DIFFERENTIAL OPERATOR WHICH EXTENDS TO A GENERATOR OF A FELLER SEMIGROUP

V. KNOPOVA

We consider an integro-differential operator

$$\begin{aligned} Lf(x) &= b(x) \cdot \nabla f(x) \\ &\quad + \int_{\mathbb{R}^d \setminus \{0\}} \Big(f(x+u) - f(x) - \nabla f(x) \cdot u 1_{|u| \le 1} \Big) N(x, du), \end{aligned}$$

defined on the space $C^2_\infty(\mathbb{R}^d)$ of twice continuously differentiable functions with vanishing at infinity derivatives. The drift $b \in \mathbb{R}^d$ is assumed to be bounded and Hölder continuous, and the Lévy-type kernel $N(x, du)$ is a sum of an α-stable like part and a lower order perturbation.

We show that under certain regularity assumptions on the kernel N one can associate with $(L, C^2_\infty(\mathbb{R}^d))$ a Feller process.

The talk is based on the on-going work with A. Kulik and R. Schilling.

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COALESCING-FRAGMENTATING WASSERSTEIN DYNAMICS ON THE REAL LINE

VITALII KONAROVSKYI

The discussion will be devoted to an interacting particle system on the real line which is similar to the Howitt-Warren flow [1, 2]. The main difference is that particles carry mass which influence their motion. Namely, it is a system of sticky-reflected Brownian particles on the real line which intuitively can be described as follows. Particles, labeled[1] by points from (0,1), start from a countable or uncountable set of massive points and move as independent Brownian motions with diffusion rates inversely proportional to their masses. When particles collide, they coalesce creating a new particle (more precisely, a set of particles occupying the same position) with mass that equals the sum of masses of the incident particles. After this, each particle experiences a drift force which makes particles split up. The drift force of a particle $u \in (0,1)$ is defined according to a given interaction potential ξ as follows:

$$\xi(u) - \frac{1}{m(u,t)} \int_{\pi(u,t)} \xi(v) dv,$$

where ξ is a right-continuous non-decreasing function, $\pi(u,t)$ is the set of all particles which share the same position with the particle u and $m(u,t) = \mathrm{Leb}(\pi(u,t))$ is its mass.

Let $X_t(u)$ denote a position of particle $u \in (0,1)$ at time t. Then the model appears as a solution to the infinite dimensional SDE with discontinuous coefficients [3, 4] (see also [5, 6, 7] for the purely coalescent case, i.e. $\xi = 0$):

$$dX_t = \mathrm{pr}_{X_t} dW_t + (\xi - \mathrm{pr}_{X_t}\xi)dt, \quad X_0 = g \tag{1}$$

in the subspace $L_2^{\uparrow}$ of non-decreasing functions from $L_2 := L_2([0,1], du)$, where pr_f is the projection in L_2 on the linear subspace of $\sigma(f)$-measurable functions and $dW.$ is an L_2-white noise. The non-decreasing right-continuous functions ξ and g are responsible for reflection between particles and their positions at the start, respectively.

Theorem 1 ([3]). (i) *(existence of solutions) For each $\delta > 0$, $g \in L_{2+\delta}^{\uparrow}$ and $\xi \in L_{\infty}^{\uparrow}$ there exists a weak solution to SDE* (1).

(ii) *(existence of a "good" modification) If g, ξ are piecewise $\frac{1}{2}$+-Hölder continuous, then there exists a random element $X = \{X(u,t),\ u \in [0,1],\ t \geq 0\}$ in the Skorohod space $D([0,1], C([0,\infty)))$ which satisfies the following properties:*

($R1$) *$X(\cdot, 0) = g$;*

($R2$) *for each $u < v$ from $[0,1]$ and $t \geq 0$, $X(u,t) \leq X(v,t)$;*

($R3$) *for all $u \in (0,1)$ the process*

$$M^X(u,t) := X(u,t) - g(u) - \int_0^t \left(\xi(u) - \frac{1}{m(u,t)} \int_{\pi(u,t)} \xi(v) dv \right) ds, \quad t \geq 0,$$

is a continuous square integrable martingale with respect to the joint filtration;

[1] The order between labels and positions of particles on the real line are preserved.

($R4$) *the joint quadratic variation of* $M^X(u,\cdot)$ *and* $M^X(v,\cdot)$ *equals*

$$[M^X(u,\cdot), M^X(v,\cdot)]_t = \int_0^t \frac{\mathbb{I}_{\{X(u,s)=X(v,s)\}}}{m_X(u,s)} ds,$$

where $m_X(u,t) := \mathrm{Leb}\{v :\ X(u,t) = X(v,t)\}$.

Moreover, the process $X(\cdot,t)$, $t \geq 0$, *is a weak solution to* (1).

Let $N(t)$ denote a number of particles at time t, that is,

$$N(t) := \|\mathrm{pr}_{X_t}\|_{HS}^2 = \left\{ \begin{array}{l} \text{a number of distinct values of} \\ \text{the right-continuous version of } X_t \end{array} \right\},$$

where $\|\cdot\|_{HS}$ denotes the Hilbert-Schmidt norm.

Theorem 2 ([8]). (i) *If* X *solves equation* (1), *then*

$$\int_0^T N(t)dt < \infty \quad a.s.$$

(ii) *If* ξ *is strictly increasing on some subinterval, then with probability 1 there exists a dense (random) subset* R *of* $[0,\infty)$ *such that* $N(t) = \infty$ *for all* $t \in R$.

The talk is based on joint work with Max von Renesse.

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DIFFUSION IN MEDIA WITH MEMBRANES AND SOME NONLOCAL PARABOLIC PROBLEMS

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Consider on plane (s,x) the strip

$$\Pi[0,T]=\{(s,x):\quad 0\le s\le T;\ -\infty<x<\infty\}$$

and the two domains $(i=1,2)$

$$S_t^{(i)}=\{(s,x)\in\Pi[0,T]:\quad 0\le s<t\le T,\ h_i(s)<x<h_{i+1}(s)\}$$

in it, where $h_j(s),\ s\in[0,T],\ j=1,2,3$ are the given functions. Put

$$I_{1s}=[h_1(s),h_2(s)),\ I_{2s}=(h_2(s),h_3(s)],\ I_s=I_{1s}\cup I_{2s},\ S_t=S_t^{(1)}\cup S_t^{(2)}.$$

Consider in $\Pi[0,T]$ two parabolic operators of the second order with bounded continuous coefficients

$$\frac{\partial}{\partial s}+L_s^{(i)}\equiv\frac{\partial}{\partial s}+\frac{1}{2}b_i(s,x)\frac{\partial^2}{\partial x^2}+a_i(s,x)\frac{\partial}{\partial x},\quad i=1,2.$$

The problem is to find a solution $u(s,x,t)$ $((s,x)\in\overline{S_t})$ of equation

$$\frac{\partial u}{\partial s}+L_s^{(i)}u=0,\quad (s,x)\in S_t^{(i)},\ i=1,2, \tag{1}$$

which satisfies the 'initial' condition

$$\lim_{s\uparrow t}u(s,x,t)=\varphi(x),\quad x\in\overline{I_t}, \tag{2}$$

two boundary conditions

$$\frac{\partial u(s,h_{2i-1}(s),t)}{\partial x}=0,\quad 0\le s<t\le T,\ i=1,2, \tag{3}$$

and two conjugation conditions

$$u(s,h_2(s)-0),t)=u(s,h_2(s)+0,t),\quad 0\le s\le t\le T, \tag{4}$$

$$q_1(s)\frac{\partial u(s,h_2(s)-0,t)}{\partial x}-q_2(s)\frac{\partial u(s,h_2(s)+0,t)}{\partial x}+$$
$$+\int\limits_{I_s}[u(s,h_2(s),t)-u(s,y,t)]\mu(s,dy)=0,\quad 0\le s\le t\le T. \tag{5}$$

The functions $q_1(s)$ and $q_2(s)$ in (5) are nonnegative and such that $q_1(s)+q_2(s)>0,\ s\in[0,T];\ \mu(s,\cdot)$ is the nonnegative measure on I_s such that for any $\delta>0$

$$\int\limits_{I_s^\delta}|y-h_2(s)|\mu(s,dy)+\mu(s,I_s\setminus I_s^\delta)<\infty,\quad s\in[0,T],$$

where $I_s^\delta=\{y\in I_s:\quad |y-h_2(s)|<\delta\}$.

The report is devoted to the study of two related questions: first, establishment of the classical solvability of the parabolic conjugation problem (1)-(5) by the boundary integral equations method (under some additional assumptions on its output data) [1, 2] and the second, construction by means of its solution of the two-parameter Feller semigroup associated with some inhomogeneous Markov process on the given region of the line. The union of these two questions represents the so-called problem on pasting together two diffusion processes given on I_{is} by their generating differential operators $L_s^{(i)}$, $i = 1, 2$ (see [3]-[7]). This problem can be also treated as the problem on construction of the mathematical model for the physical phenomenon of diffusion in medium with moving membranes [2].

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RANDOM OPERATORS RELATED TO A HARRIS FLOW

IA. A. KORENOVSKA

Definition 1. ([2]) A Harris flow with covariance function Γ is a family $\{x(u,\cdot),\ u\in\mathbb{R}\}$ of Brownian martingales with respect to the joint filtration such that

(1) for any $u\in\mathbb{R}\quad x(u,0)=u$;
(2) for every $u_1,u_2\in\mathbb{R},\ u_1\le u_2,\ t\ge 0\quad x(u_1,t)\le x(u_2,t)$;
(3) for any $u_1,u_2\in\mathbb{R},\ t\ge 0$

$$< x(u_1,\cdot),x(u_2,\cdot)>(t)=\int_0^t \Gamma(x(u_1,s)-x(u_2,s))ds.$$

Let Γ be a real continuous nonnegative definite function with spectral distribution which is not of the pure jump type. It is assumed throughout that Γ is Lipshitz outside each interval $(-c,c),\ c>0,\ \Gamma(0)=1$, and there exist $\varepsilon\in(0;2),\ a>0$ such that for any $u\in\mathbb{R},\ |u|\le a$,

$$1-\Gamma(u)\ge |u|^{2-\varepsilon}.$$

It is proved in [1] that under these conditions the Harris flow exists, and for each $t>0$ $x(\mathbb{R},t)$ is a countable set, which means that particles in the Harris flow coalesce.

Define a family $\{T_t,\ t\in[0;1]\}$ of random operators in $L_2(\mathbb{R})$ which describe shifts of functions along the Harris flow, i.e. for fixed $t\in[0;1]$

$$(T_tf)(u)=f(x(u,t))$$

where $f\in L_2(\mathbb{R}),\ u\in\mathbb{R}$.

Remark 1. T_t is a strong random operator ([3]) in $L_2(\mathbb{R})$ for any $t\in[0;1]$ (see [2]). Consequently, T_tf is $L_2(\mathbb{R})$-valued random variable for every $f\in L_2(\mathbb{R}),\ t\in[0;1]$.

For fixed $f\in L_2(\mathbb{R})$ we consider $L_2(\mathbb{R})$-valued random process

$$T^{(f)}=\{T_tf,\ t\in[0;1]\}$$

and investigate its properties.

Theorem 1. *If $T^{(f)}$ has càdlàg modification then $f\in L_2(\mathbb{R})\cap C(\mathbb{R})$.*

Theorem 2. *If $f\in L_2(\mathbb{R})\cap C(\mathbb{R})$ then $T^{(f)}$ is continuous on $[0;1]$.*

Under condition $f\in L_2(\mathbb{R})\cap C(\mathbb{R})$ the action of random shift operators along the Harris flow on f has a continuous trajectory. To study a change in time of a family of functions under random operators T_t one need to know when $T^{(f)}$ has a continuous trajectory simultaneously for f from the certain family.

Theorem 3. *Let a family of functions $\Phi\subset C(\mathbb{R})$ be such that*

$$\lim_{n\to\infty}\sup_{f\in\Phi}\int_{|u|>n} f^2(u)du=0,$$

$$\forall\varepsilon>0 \quad \exists\delta(\varepsilon)>0 \quad \forall u_1,u_2\in\mathbb{R},\ |u_1-u_2|<\delta(\varepsilon) \qquad \sup_{f\in\Phi}|f(u_1)-f(u_2)|<\varepsilon.$$

Then almost surely for any $f\in\Phi$ the random process $T^{(f)}$ is continuous on $[0;1]$.

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NOISE SENSITIVITY OF LÉVY DRIVEN SDE'S: ESTIMATES AND APPLICATIONS

TETIANA KOSENKOVA

The topic of this talk is induced by the following question: whether the deviation between the solutions of two different Lévy driven SDE's can be controlled in terms of the characteristics of the underlying Lévy processes? In the case of SDE's with additive noise we give the estimate for the deviation between the solutions in terms of the coupling distance for Lévy measures, which is based on the notion of the Wasserstein distance. In case of Lévy-type processes, whose characteristic triplets are state dependent, we exploit the fact that every Lévy kernel can be obtained by means of a certain infinite Lévy measure and the transform function. And under an appropriate set of conditions on the state dependent characteristic triplet the Lévy-type process can be described as a strong solution to a Lévy driven SDE with multiplicative noise. The estimate of the deviation between two Lévy-type processes is given in terms of transportation distance between the Lévy kernels, which uses the transform functions of the kernels. Such estimates can be applied to the analysis of the low-dimensional conceptual climate models with paleoclimate data.

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STOCHASTIC PROCESSES FROM THE SPACES $\mathbf{F}_\psi(\Omega)$. CONDITIONS FOR THE WEAK CONVERGENCE

YURII KOZACHENKO, YURII MLAVETS

Definition 1. [1] We say that the condition **H** is fulfilled for the Banach space of random variables $B(\Omega)$, if there exists an absolute constant C_B such that for any centered and independent random variables $\xi_1, \xi_2, \ldots, \xi_n$ from $B(\Omega)$, the following is true:

$$\left\| \sum_{i=1}^{n} \xi_i \right\|^2 \leq C_B \sum_{i=1}^{n} \|\xi_i\|^2 .$$

The constant C_B is called a scale constant for the space $B(\Omega)$. For space $\mathbf{F}_\psi(\Omega)$ we shall denote the constants $C_{\mathbf{F}_\psi(\Omega)}$ as C_ψ.

Let $X = \{X(t),\ t \in T\}$ be a stochastic process from the space $\mathbf{F}_\psi(\Omega)$, $EX(t) = 0$. Let the condition **H** be fulfilled for this space.

Assume that compact pseudometric space (T, ρ_ψ), $\rho_\psi(t,s) = \|X(t) - X(s)\|_\psi$ is separable and the process $X = \{X(t),\ t \in T\}$ is separable as well. Let $X_k(t)$, $k = 1, 2, \ldots, n$ be independent copies of $X(t)$. Consider a stochastic process $Y_n(t) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} X_k(t)$. By Definition (1) we have $\|Y_n(t) - Y_n(s)\|_\psi^2 \leq C_\psi \frac{1}{n} \sum_{k=1}^{n} \|X_k(t) - X_k(s)\|_\psi^2 = C_\psi \rho_\psi^2(t,s)$.

The pseudometric space (T, ρ_ψ) is separable and the processes $Y_n(t)$ are separable in this space.

Theorem 1. [1] *If the following condition holds*

$$\hat{\varepsilon}_0 = \sup_{t,s \in T} \|X(t) - X(s)\|_\psi < \infty,$$

and for any $\tau > 0$

$$\int_0^\tau \kappa_\psi(\tilde{N}(u))du < \infty,$$

where $\kappa_\psi(n)$ *is the* M*-characteristic of the space* $\mathbf{F}_\psi(\Omega)$, $\tilde{N}(\varepsilon)$ *is the metric massiveness of the space* (T, ρ_ψ), *then* $Y_n(t)$ *converge weakly in* $\mathbf{C}(T, \rho_\psi)$ *to the Gaussian process* $X_\infty(t)$ *such that* $EX_\infty(t) = 0$, $EX_\infty(t)X_\infty(s) = EX(t)X(s)$.

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REAL STATIONARY GAUSSIAN PROCESSES WITH STABLE CORRELATION FUNCTIONS

YURII KOZACHENKO, MARYNA PETRANOVA

Real stationary processes with stable correlation functions, distribution of some functionals of these processes and some of their properties are considered.

We continue the study of work [3], which dealt with complex Gaussian processes with a stable correlation function. Stationary processes with a stable correlation function are considered, in particular, the distribution of some functionalities from these processes and some of their properties. For other processes, similar tasks were considered in [2, 1, 4, 5].

Definition 1. The real stationary Gaussian process $X_\alpha = \{X_\alpha(t), t \in \mathbf{R}\}$, $0 < \alpha \le 2$, such that $EX_\alpha(t) = 0$, $\rho_\alpha(h) := EX_\alpha(t+h)X_\alpha(t) = B^2 exp\{-d|h|^\alpha\}$, $\alpha > 0, d > 0$ is called a real Gaussian stationary process with a stable correlation function.

Theorem 1. *Let X_α be a real separable Gaussian stationary process with a stable correlation function. Then for any $-\infty < a < b < +\infty$, $0 < \theta < 1$, $\beta < \min\left(1, \frac{\alpha}{2}\right)$, $\epsilon > 0$ the inequality is fulfilled:*

$$P\left\{\sup_{t\in[a,b]} |X(t)| > \epsilon\right\} \le exp\left\{-\frac{\epsilon^2(1-\theta)^2}{2B^2}\right\} \cdot 2^{1/\beta - 1}\left(\frac{(b-a)(\sqrt{2d})^{2/\alpha}}{\theta^{2/\alpha}\left(1-\frac{2\beta}{\alpha}\right)^{1/\beta}} + 1\right) \tag{1}$$

Theorem 2. *Let $X_\alpha = \{X_\alpha(t), t \in \mathbf{R}\}$ be a real Gaussian stationary process with a stable correlation function (see Definition 1), $C = \{C(t), t \ge 0\}$ - monotonically increasing function, such that $C(t) \ge 1, t \ge 0$ and $C(t) \to \infty$ by $t \to \infty$; $b_0, b_1, b_2, ..., b_k$ - such a sequence that $b_0 = 0, b_k < b_{k+1}$, and $b_k \to \infty$ by $k \to \infty$, $r_0, r_1, r_2, ..., r_k$ such a sequence that $r_k > 1$ and $\sum_{k=0}^{\infty} \frac{1}{r_k} = 1$, $C_k = C(b_k), k = 0, 1, 2...$ and the following conditions be fulfilled:*

$$\sum_{k=0}^{\infty} \frac{r_k}{C_k^2} < \infty,$$

$\sum_{k=0}^{\infty} \frac{1}{r_k}(b_{k+1} - b_k)^\gamma < \infty$, where $\gamma \in (0,1)$. Then for any $0 < \theta < 1$, $\epsilon > 0$ the following inequality holds:

$$P\left\{\sup_{t\ge 0} \frac{|X_\alpha(t)|}{C(t)} > \epsilon\right\} \le 2^{\frac{4}{\alpha}-1} exp\left\{-\frac{\epsilon^2(1-\theta)^2}{2B^2 \sum_{k=0}^{\infty} \frac{r_k}{C_k^2}}\right\} \cdot$$
$$\cdot exp\left\{\frac{1}{\theta^{\gamma/2} \cdot \gamma^\gamma} \cdot (\sqrt{2d})^{\frac{2\gamma}{\alpha}} \cdot 2^{\frac{4\gamma}{\alpha}} \sum_{k=0}^{\infty} \frac{1}{r_k}(b_{k+1} - b_k)^\gamma\right\} \tag{2}$$

Definition 2. Random process $X(t)$, $t \in [a,b]$, is called differentiable in mean square, if there exists a limit (in mean square)

$$l.i.m._{h\to 0}\frac{X(t+h)-X(t)}{h} = X'(t)$$

If this limit exists, then $X'(t)$ is called a mean square derivative of the process $X(t)$.

Theorem 3. *Let $X_\alpha(t)$, $t \in [a,b]$, be a stationary process with stable correlation function (not necessarily Gaussian). Then for $0 < \alpha < 2$ the mean square derivatives do not exist, and for $\alpha = 2$ the derivative exists and*

$$EX_2'(t)X_2'(s) = B^2 exp\left\{-d(t-s)\right\} \cdot \left(4d^2 \cdot (t-s)^2 + 2d\right)$$

i.e. $X_2'(t)$ is a stationary process with correlation function

$$EX_2'(t+\tau)X_2'(t) = B^2 exp\left\{-d|\tau|^2\right\} \cdot \left(4d^2 \cdot \tau^2 + 2d\right) \tag{3}$$

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BAXTER TYPE THEOREM FOR GENERALIZED GAUSSIAN PROCESSES WITH INDEPENDENT VALUES

S. M. KRASNITSKIY, O. O. KURCHENKO

Let K be the space of compactly supported infinitely differentiable functions on the interval $(-\infty, \infty)$. The covariation functional $B(\varphi, \psi)$ of a generalized random process ξ with independent values on K ($\xi = (\xi, \varphi)$, $\varphi \in K$) admits representation

$$B(\varphi, \psi) = \int_{-\infty}^{\infty} \sum_{k,j \geq 0} B_{kj}(x) \frac{d^k \varphi(x)}{dx^k} \cdot \frac{d^j \psi(x)}{dx^j}\, dx, \quad \varphi, \psi \in K, \tag{1}$$

in which $B_{k,j}(x)$ are continuous on the interval $(-\infty, \infty)$ functions, and in each bounded interval only the finite number of functions $B_{k,j}(x)$ are other than identical zero [1]. We will consider the process ξ on the set $K\,([0,1])$ which is the subspace of K, formed by the functions $\varphi \in K$ with supports in the interval $(0,1)$, and as functions $B_{k,j}(x)$ for this restriction ξ we take constants. In this case, using the integration by parts and taking into account the symmetry of the real bilinear functional $B(\varphi, \psi)$ the expression (1) as a functional on $K\,([0,1])$ admits the representation

$$B(\varphi, \psi) = \sum_{j=0}^{N} c_j \frac{d^j \varphi(x)}{dx^k} \cdot \frac{d^j \psi(x)}{dx^j}\, dx, \quad \varphi, \psi \in K(0,1), \tag{2}$$

where c_j are (constant) coefficients, $j = 0, \ldots, N; N < +\infty$.

We note that the partial case of the process ξ is a well–known generalized "white noise" process, which has a covariation functional that coincides with the summand on the right side (2) for $k = j = 0$ (and all $\varphi, \psi \in K$).

We assume that $B(\varphi, \psi)$ for $\varphi, \psi \in K(0,1)$ is represented exactly in the form (2), and the process ξ is Gaussian with zero mean value. As in [2], the expression

$$S_n(\xi) = \sum_{k=0}^{b(n)-1} (\xi, \alpha_{k,n})^2 ,$$

where $\{b(n), n = 1, 2, \ldots\}$ is a non-decreasing integer numbers sequence, $b(n) \xrightarrow[n \to \infty]{} \infty$, and $\alpha_{k,n} = \alpha_{k,n}(\cdot) \in K$ is a function with a support in the interval $\left(\frac{k}{b(n)}, \frac{k+1}{b(n)}\right)$, we call a Baxter sum.

Our report explicitly presents a family of functions $\{\alpha_{k,n}\}$ such that for the Baxter sum of the process ξ we have a limit relation

$$\lim_{n \to \infty} S_n(\xi) = c_N, \tag{3}$$

where the convergence takes place in square mean. If the series $\sum_{n=1}^{\infty} (b(n))^{-1}$ is convergent, then we have also the almost sure convergence in (3).

We note that the coefficients $c_0, c_1, \ldots, c_{N-1}$ cannot be defined by the limit values of the Baxter sums of the process ξ with $c_N \neq 0$. The latter follows from the results of work [3].

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CONVERGENCE OF SKEW BROWNIAN MOTIONS WITH LOCAL TIMES AT SEVERAL POINTS THAT ARE CONTRACTED INTO A SINGLE ONE

IVAN H. KRYKUN

We consider the skew Brownian motion as a solution of the stochastic equation with local times at N points and with coefficients depending on the parameter n

$$\xi_n(t) = \beta_1(n) L^{\xi_n}(t,0) + \sum_{i=2}^{N} \beta_i(n) L^{\xi_n}(t, a_i(n)) + w(t), \quad t \in [0,T]. \tag{1}$$

We will study the question about the convergence of solutions of the stochastic equation (1) under the condition that, as the parameter $n \to \infty$, the coefficients of the local times $\beta_i(n)$ tend, in this case, to their limit values β_i $(i = 1, ..., N)$, respectively, and the points $a_i(n)$ tend to 0 $(i = 2, ..., N)$.

For the skew Brownian motion (1), we introduce the following condition.

Condition (I).

- $|\beta_i(n)| < 1$ for all n and $i = 1, ..., N$.
- There exist constants β_i such that $|\beta_i| < 1$, $i = 1, ..., N$, and

$$\lim_{n\to\infty} \beta_i(n) = \beta_i, \quad i = 1, ..., N.$$

- $a_i(n) > 0$ for all n, $i = 2, ..., N$.
- $a_i(n) \neq a_j(n)$ for $i \neq j$ for all n, $i, j = 2, ..., N$.
- For $i = 2, ..., N$ condition $\lim\limits_{n\to\infty} a_i(n) = 0$ holds.

Theorem. [1, Theorem 1]. *Let condition (I) be satisfied for Eq.* (1). *Then the convergence of the processes of the skew Brownian motion* (1) *to the limit process*

$$\xi(t) = \gamma L^{\xi}(t,0) + w(t), \quad t \in [0,T],$$

holds in mean uniformly in time, as $n \to \infty$. The coefficient γ of the local time of the limit process $\xi(t)$ is given by the formula

$$\gamma = \frac{\prod\limits_{i=1}^{N}(1+\beta_i) - \prod\limits_{i=1}^{N}(1-\beta_i)}{\prod\limits_{i=1}^{N}(1+\beta_i) + \prod\limits_{i=1}^{N}(1-\beta_i)}. \tag{2}$$

Let us denote a hyperbolic tangent of x by $\tanh x$ and an areatangent (inverse hyperbolic tangent) of x by $\mathrm{artanh} x$.

Remark 1. The coefficient γ of the local time of a limit process, given by formula (2), also can be found by the equivalent formula

$$\gamma = \tanh\Big(\sum_{i=1}^{N} \operatorname{artanh} \beta_i\Big), \tag{3}$$

Example. *Consider the skew Brownian motion with local times at two points that are contracted into a single one:*

$$\xi_n(t) = \beta_1(n)L^{\xi_n}(t,0) + \beta_2(n)L^{\xi_n}(t,a(n)) + w(t), \quad t \in [0,T].$$

Let it satisfy the condition (I). What is the limit process?

According to theorem, the limit process is

$$\xi(t) = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}L^{\xi}(t,0) + w(t), \quad t \in [0,T].$$

Remark 2. Similar issues are considered in [2], [3].

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CONSISTENCY OF TOTAL LEAST SQUARES ESTIMATOR IN VECTOR LINEAR ERRORS-IN-VARIABLES MODEL WITH INTERCEPT

ALEXANDER KUKUSH

An implicit linear errors-in-variables model is considered. Within this observation model, the true points are nonrandom, belong to a linear manifold (of known dimension) in a Euclidean space, and are observed with additive errors. The total error covariance matrix is proportional to the identity matrix, with unknown factor of proportionality. The normality of errors is not assumed, and it is not demanded that the error vectors are independent and identically distributed.

The orthogonal regression estimator (ORE) of the manifold is studied, which in the case of normal errors coincides with the maximum likelihood estimator. Sufficient conditions are presented for the consistency and strong consistency of the ORE, as the sample size tends to infinity.

The results are applied to an explicit linear errors-in-variables model with intercept, where covariates are vectors and the response is a vector as well. The latter model is embedded into the implicit model with intercept, and as a result the ORE of the manifold yields the total least squares (TLS) estimator of the regression function. The conditions for consistency of the TLS estimator of intercept and of matrix regression parameter are stated separately.

A particular case, where the underlying manifold is a hyperplane and the corresponding explicit model describes multiple regression with scalar response, is investigated in [1].

Results of the talk are joint with M.S. student Oleksandr Dashkov (Kyiv).

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ASYMPTOTIC ANALYSIS OF MULTI-SCALE LÉVY DRIVEN STOCHASTIC SYSTEMS

ALEXEI KULIK

A new method will be presented, well suitable for the study of averaging/diffusion approximation phenomena in multi-scale stochastic systems, including the so called *fully coupled* systems, which are physically most relevant and technically most involved. This *time delay method*, introduced in [1], does not use corrector terms and thus does not require auxiliary Poisson equations to be solved, which makes it well applicable to the systems with complicated local structure, including the systems driven by Lévy noises. In the talk, a diffusion approximation theorem for a fully coupled Lévy driven stochastic system will be presented, together with ergodicity, regularity, and sensitivity issues, substantially used in the entire construction. Particular examples of multi-scale stochastic versions of the Cucker-Smale flocking model and the non-linear friction model will be discussed.

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FRACTIONAL PEARSON DIFFUSIONS AND CONTINUOUS TIME RANDOM WALKS

N. LEONENKO

We define fractional Pearson diffusions [5,7,8] by non-Markovian time change in the corresponding Pearson diffusions [1,2,3,4] inspiring ideas of I.I. Gikhman and A.V.Skorokhod. They are governed by the time-fractional diffusion equations with polynomial coefficients depending on the parameters of the corresponding Pearson distribution. We present the spectral representation of transition densities of fractional Pearson diffusions, which depend heavily on the structure of the spectrum of the infinitesimal generator of the corresponding non-fractional Pearson diffusion. Also, we present the strong solutions of the Cauchy problems associated with heavy-tailed fractional Pearson diffusions and the correlation structure of these diffusions [6].

We define the correlated continuous time random walks (CTRWs) that converge to fractional Pearson diffusions (fPDs) [9]. The jumps in these CTRWs are obtained from Markov chains through the Bernoulli urn-scheme model and Wright-Fisher model. The jumps are correlated so that the limiting processes are not Lévy but diffusion processes with non-independent increments. The waiting times are selected from the domain of attraction of a stable law.

This is a joint work with M. Meerschaert (Michigan State University, USA), I. Papic (University of Osijek, Croatia), N. Suvak (University of Osijek, Croatia) and A. Sikorskii (Michigan State University and Arizona University, USA).

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SURVIVAL ANALYSIS BY MIXTURES WITH VARYING CONCENTRATIONS

ROSTYSLAV MAIBORODA

Random right-censoring is a standard probabilistic model for survival data description. The Kaplan-Meyer estimator is usually used for nonparametric estimation of survival time distribution in this model. We consider a modification of this estimator for the case when the observed data were obtained from a mixture of components with different distributions. Such modifications were considered in the papers [1, 4] in the case of known varying concentrations (mixing probabilities) in the mixture. (See [3] on mixture models with varying concentrations).

Let us present a motivating example of problem in which these estimates can be applied.

Let there be medical statistics data on the course of an ontological disease for some set of patients. We are interested in the distribution of duration of remission (ξ) for this disease. Here remission is the interval between the surgery on the tumor removal and the relapse. Note that the remission duration can be from several weeks to several decades.

Some patients may fell out of our sight at the remission interval, so the data are randomly right censored. I.e. we observe $\xi^* = \min(\xi, C)$ and $\delta = \mathbf{1}\{\xi < C\}$, where C is the censoring time (censor), δ is the indicator of non-censoring. Suppose that we have got such data on many patients for a long time (say, 50 years). Earlier it was assumed that all the considered patients have the same disease. But 5 years ago it was observed that such tumors can be caused by two different genetic causes (mutations in different loci). So, from the modern point of view, there are two diseases types (A and B) of which any our patient could suffer.

The true type of the disease can be identified by a DNA-analysis of the tumor. This can be done for the modern patients (patients of last 5 years). But we can't estimate, say the probability that the remission duration for type-A patients will exceed 10 years. We didn't observe the modern patients for 10 years!

We have a lot of old data (on patients cured 50 years ago). But it is impossible to identify uniquely the type of these patients' disease. On the other hand, the medical records for them contain information on some symptoms of their illness. Comparing this information with the symptoms of the modern patients, we can estimate probabilities that a patient from the old records had a disease of type A (or B).

So, in this example we deal with a mixture of two components (A and B) and the probability of a subject (patient) to belong to a given component is known (estimated). At the same time the data on the variable of interest (the remission duration) are censored.

Our aim is to utilize such information for estimation of the distribution of remission duration for the disease of a specified type.

In the presentation we will discuss the asymptotic behavior of the modified Kaplan-Meyer estimators for mixtures and their application for hypotheses testing.

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CONES GENERATED BY RANDOM POINTS ON HALF-SPHERES AND CONVEX HULLS OF POISSON POINT PROCESSES

ALEXANDER MARYNYCH

Let $d \geq 1$ be a fixed positive integer and let $U_1, U_2, \ldots$ be independent random points distributed according to the uniform distribution on the d-dimensional *upper half-sphere*

$$\mathbb{S}^d_+ := \{(x_0, x_1, \ldots, x_d) \in \mathbb{R}^{d+1} \colon x_0^2 + x_1^2 + \ldots + x_d^2 = 1,\ x_0 \geq 0\}.$$

We are interested in the random convex cone in $\mathbb{R}^{d+1}$ defined as the *positive hull* of $U_1, \ldots, U_n$, $n \geq d+1$, that is

$$C_n = \operatorname{pos}\{U_1, \ldots, U_n\} := \{\alpha_1 U_1 + \ldots + \alpha_n U_n \colon \alpha_1, \ldots, \alpha_n \geq 0\}.$$

The random cone, or, more precisely, the random spherical polytope $C_n \cap \mathbb{S}^d_+$, has been studied by Bárány et al. [1]. Some of their results concern the expected f-vector of C_n, that is, the expected number $\mathbb{E} f_k(C_n)$ of k-dimensional faces of C_n, $k \in \{1, \ldots, d\}$. In particular, by Theorem 7.1 in [1] the expected number of one-dimensional faces of C_n (or, equivalently, vertices of $C_n \cap \mathbb{S}^d_+$) satisfies

$$\lim_{n \to \infty} \mathbb{E} f_1(C_n) = C(d) \pi^{d+1} \left(\frac{2}{\omega_{d+1}} \right)^{d+1} \omega_d, \tag{1}$$

where a constant $C(d)$ is given in form of a multiple integral; see [1, Equation (22)] and ω_d is the $(d-1)$-dimensional Hausdorff measure (surface area) of the unit sphere $\mathbb{S}^{d-1} \subset \mathbb{R}^d$, that is

$$\omega_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}.$$

This surprising result, which says that the expected number of one-dimensional faces of C_n remains bounded as the size of the sample tends to infinity, is the starting point for our work.

Our first main result is a weak limit theorem for the sections of the random cones $(C_n)_{n \in \mathbb{N}}$ with the tangent hyperplane of the half-sphere at its north pole. Let $T_n : \mathbb{R}^{d+1} \to \mathbb{R}^{d+1}$ be the mapping defined by

$$T_n(x_0, x_1, \ldots, x_d) := (n x_0, x_1, \ldots, x_d).$$

Let H_1 be the hyperplane $\{x_0 = 1\}$ in $\mathbb{R}^{d+1}$. Note that H_1 is tangent to the half-sphere $\mathbb{S}^d_+$ at its north pole. Let e_0 be the unit vector $(1, 0, \ldots, 0) \in \mathbb{R}^{d+1}$ pointing to the north pole. To describe the limit, take some $\gamma > 0$, $c > 0$, and let $\Pi_{d,\gamma}(c)$ be a Poisson point process on $\mathbb{R}^d \backslash \{0\}$ whose intensity measure is absolutely continuous with respect to the Lebesgue measure and whose density function is given by

$$x \mapsto \frac{c}{\omega_{d+\gamma}} \frac{1}{\|x\|^{d+\gamma}}, \quad x \in \mathbb{R}^d \backslash \{0\}, \tag{2}$$

where $\|x\|$ is the Euclidean norm of x. Note that the number of points of $\Pi_{d,\gamma}(c)$ outside any ball centered at the origin having strictly positive radius is almost surely finite (because the intensity is integrable near ∞), while the number of points inside any such ball is infinite with probability one (because the integral of the intensity over such balls diverges). We denote by $\operatorname{conv}\Pi_{d,\gamma}(c)$ the convex hull of all points of $\Pi_{d,\gamma}(c)$. Even though $\Pi_{d,\gamma}(c)$ almost surely consists of infinitely many points, the random convex set $\operatorname{conv}\Pi_{d,\gamma}(c)$ turns out to be almost surely a polytope. The next theorem identifies the weak limit of the rescaled random polytopes $(T_nC_n \cap H_1) - e_0$ in terms of a Poisson point process of the type just discussed.

Theorem 1. *As $n \to \infty$, the random polytopes $(T_nC_n \cap H_1) - e_0$ converge in distribution to $\operatorname{conv}\Pi_{d,1}(2)$ on the space of compact convex subsets of $\mathbb{R}^d$ endowed with the Hausdorff metric.*

From Theorem 1 we shall derive the following result on the distributional convergence of the f-vector of the random spherical polytope $C_n \cap \mathbb{S}^d_+$. Note that $f_k(C_n \cap \mathbb{S}^d_+) = f_{k+1}(C_n)$.

Theorem 2. *As $n \to \infty$, we have that*

$$\left(f_0(C_n \cap \mathbb{S}^d_+), \dots, f_{d-1}(C_n \cap \mathbb{S}^d_+)\right) \xrightarrow{\mathrm{d}} \left(f_0(\operatorname{conv}\Pi_{d,1}(2)), \dots, f_{d-1}(\operatorname{conv}\Pi_{d,1}(2))\right),$$

where $\xrightarrow{\mathrm{d}}$ denotes convergence in distribution.

We shall argue also that the *expected* f-vector of the spherical random polytope $C_n \cap \mathbb{S}^d_+$ converges to that of $\operatorname{conv}\Pi_{d,1}(2)$. Even more generally, we shall prove the convergence of moments of all orders. This generalizes the results from [1] discussed above and answers – in an extended form – a question raised in [1, Section 9].

Theorem 3. *For every $k \in \{1, \dots, d\}$ and every $m \in \mathbb{N}$ we have*

$$\lim_{n\to\infty} \mathbb{E} f_k^m(C_n) = \lim_{n\to\infty} \mathbb{E} f_{k-1}^m(C_n \cap \mathbb{S}^d_+) = \mathbb{E} f_{k-1}^m(\operatorname{conv}\Pi_{d,1}(2)).$$

The next theorem deals with the solid angle of C_n. Let $\bar\sigma$ be the d-dimensional spherical Lebesgue measure on the unit sphere $\mathbb{S}^d \subset \mathbb{R}^{d+1}$ normalized such that $\bar\sigma(\mathbb{S}^d) = 1$. The *solid angle* $\alpha(C_n)$ of the convex cone C_n is defined by

$$\alpha(C_n) := \bar\sigma(C_n \cap \mathbb{S}^d).$$

Theorem 4. *As $n \to \infty$, we have that*

$$n\left(\frac{1}{2} - \alpha(C_n)\right) \xrightarrow{\mathrm{d}} \frac{1}{\omega_{d+1}} \int_{\mathbb{R}^d \setminus \operatorname{conv}\Pi_{d,1}(2)} \frac{\mathrm{d}v}{\|v\|^{d+1}}.$$

This talk is based on a recent joint work [2] with Z. Kabluchko, D. Temesvari and C. Thäle.

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STATISTICAL CHALLENGES IN NEUROSCIENCE: HOW TO DEAL WITH DEGENERATE NOISE IN MULTIDIMENSIONAL SDES

ANNA MELNYKOVA

When the dimensionality of the Brownian motion of the process is lower than the dimensionality of the process itself, we face a very specific type of stochastic processes. Under certain conditions, despite the singular diffusion matrix, the noise is still propagated through all the system: this phenomena leads us to a class of hypoelliptic diffusions.

They often arise in modeling of neuronal activity: for example, it is proved in Ditlevsen and Löcherbach (2017) that the mean-field limit of Hawkes processes, describing the interactions of multiple populations of neurons, is approximated by a stochastic hypoelliptic diffusion. Further, models of this type are also used to describe a firing mechanism of a neuron: see, for example, Leon and Samson (2017).

To be more precise, we restrict our attention to the most simple, two-dimensional case. Consider SDE of the form:

$$\begin{cases} dX_t = a_1(X_t, Y_t)dt \\ dY_t = a_2(X_t, Y_t)dt + b(X_t, Y_t)dW_t, \end{cases} \tag{1}$$

where $(X_t, Y_t)^T \in \mathbb{R} \times \mathbb{R}$, $(a_1(X_t, Y_t),\ a_2(X_t, Y_t))^T$ is the drift term, $(0, b(X_t, Y_t))^T$ is the diffusion coefficient, (dW_t) is a standard Brownian motion defined on some probability space.

The goal of this talk is to discuss several problems assosiated with hypoelliptic system (1): first, we bring to the light a problem of a numerical approximation of such processes. This issue is tightly bounded with a parametric inference for discretely observed processes and is treated, under assumptions of ergodicity and hypoellipticity, by Samson and Thieullen (2012), Ditlevsen and Samson (2017), Melnykova (2018).

In more global sense, we are interested in how good this model can fit some specific data — for example, intracellular recordings of neuronal activity. This, among other things, includes developing statistical tests for estimating the dimensionality of the noise (see, in particular, Jacod et al. (2013)). We briefly discuss the perspectives, and conclude our study with some numerical examples.

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LARGE DEVIATIONS OF RESIDUAL CORRELOGRAM AS ESTIMATOR OF THE NOISE UNKNOWN COVARIANCE FUNCTION

K. K. MOSKVYCHOVA

Consider the observation model

$$X(t) = g(t, \theta) + \varepsilon(t), \quad t \geq 0,$$

where $g : (-\gamma, \infty) \times \Theta_\gamma \to \mathbb{R}$ is a sufficiently smooth function depending on the unknown parameter $\theta = (\theta_1, ..., \theta_q) \in \Theta \subset \mathbb{R}^q$, Θ is a bounded open convex set, $\Theta_\gamma = \bigcup\limits_{\|a\|<1} (\Theta + \gamma a)$, $\gamma > 0$ is some number.

I. $\varepsilon = \{\varepsilon(t),\ t \in \mathbb{R}\}$ is a mean-square and almost sure continuous stationary Gaussian process defined on the probability space $(\Omega, \mathfrak{F}, P)$, $\mathsf{E}\varepsilon(0) = 0$, with covariance function $B(\cdot) \in L_1(\mathbb{R})$.

The LSE of the parameter $\theta \in \Theta$ is defined as the random vector $\widehat{\theta}_T = (\widehat{\theta}_{1T}, \ldots \widehat{\theta}_{qT}) \in \Theta^c$ with the property

$$Q_T(\widehat{\theta}_T) = \min_{\tau \in \Theta^c} Q_T(\tau), \quad Q_T(\tau) = \int\limits_0^T [X(t) - g(t, \tau)]^2 dt.$$

As an estimator of B tied to the estimator $\widehat{\theta}_T$ of the nuisance parameter θ we take the residual correlogram constructed by residuals $\widehat{X}(t) = X(t) - g(t, \widehat{\theta}_T)$, $t \in [0, T + H]$, namely:

$$B_T(z, \widehat{\theta}_T) = T^{-1} \int\limits_0^T \widehat{X}(t + z)\widehat{X}(t)dt, \quad z \in [0, H],$$

$H > 0$ is a fixed number.

Let $d_T(\theta) = diag(d_{iT}(\theta), i = \overline{1, q})$, $d_{iT}^2(\tau) = \int\limits_0^T (\frac{\partial}{\partial \tau_i} g(t, \tau))^2 dt$.

Write also $f_0 = \max\limits_{\lambda \in \mathbb{R}} f(\lambda) < \infty$, where $f(\lambda)$, $\lambda \in \mathbb{R}$, is a spectral density of ε.

II. There exist numbers $c_0 > 0$, $c_1 > 0$ such that for any $\theta \in \Theta$ and $u, v \in U_T(\theta) = d_T(\theta)(\Theta^c - \theta)$ for $T > T_0$ (T_0 doesn't depend on θ)

$$c_0\|u - v\|^2 \leq \int\limits_0^T \left(g(t, \theta + d_T^{-1}(\theta)u) - g(t, \theta + d_T^{-1}(\theta)v)\right)^2 dt \leq c_1\|u - v\|^2.$$

Introduce pseudometric

$$\sqrt{\rho}(z_1, z_2) = \Big(\int\limits_{-\infty}^{\infty} f^2(\lambda) \sin^2 \frac{\lambda(z_1 - z_2)}{2} d\lambda \Big)^{1/4}, \quad z_1, z_2 \in \mathbb{R}.$$

Denote by $N_{\sqrt{\rho}}(\varepsilon) = N_{\sqrt{\rho}}([0,1],\varepsilon)$, $\varepsilon > 0$, and $H_{\sqrt{\rho}}(\varepsilon) = \ln N_{\sqrt{\rho}}(\varepsilon)$ metric massivness and metric entropy of interval $[0,1]$ with respect to pseudometric $\sqrt{\rho}$.

Let also $c(r) = -r^{-2}\ln(1-r) - r^{-1}$, $r \in (0,1)$, $r^* \simeq 0.898187$ is the only solution of the equation $-\ln(1-r) - r = 2\ln 2$. And

$$D(r) = \left(\frac{\pi c(r)}{\ln 2}\right)^{1/2} \left(98\|f\|_2 + 16\|f\|_2^{1/2} \int_0^{\varepsilon(\sqrt{\rho})} \ln(1 + N_{\sqrt{\rho}}(\varepsilon))d\varepsilon\right),$$

where $\varepsilon(\sqrt{\rho}) = \sup\limits_{z_1,z_2\in H} \sqrt{\rho}(z_1,z_2) \leq \|f\|_2^{1/2} = \left(\int\limits_{-\infty}^{\infty} f^2(\lambda)d\lambda\right)^{1/4}$.

III.
$$\int\limits_{0+} H_{\sqrt{\rho}}(\varepsilon)d\varepsilon < \infty.$$

Theorem 1. *Under conditions* **I**, **II**, *and* **III** *for any*

$$a < a_0 = \frac{1}{16\pi f_0(1+q)} \cdot \frac{c_0}{c_1}(1 \wedge \frac{1}{2B(0)}) \wedge \frac{1}{D(r^*)}$$

there exists a constant A such that for $T > T_0$, $R > R_0$

$$\mathsf{P}\Big\{T^{1/2} \sup_{z\in[0,H]} |B_T(z,\widehat{\theta}_T) - B(z)| \geq R\Big\} \leq A\exp\{-aR\}.$$

In the proof of Theorem 1 we use in particular results of the papers [1, 2] and monograph [3].

Corollary 1. *Let $\gamma \in [0,1/2)$, $h > 0$ are some numbers, $R = hT^{1/2-\gamma}$. Then under conditions of Theorem 1 for $T > T_0 \vee \{R_0/h\}^{(1/2-\gamma)^{-1}}$*

$$\mathsf{P}\Big\{\sup_{z\in[0,H]} |B_T(z,\widehat{\theta}_T) - B(z)| \geq hT^{-\gamma}\Big\} \leq A\exp\left\{-ahT^{1/2-\gamma}\right\}.$$

Corollary 2. *Let $h > 0$ is some number, $R = h\ln T$. Then under conditions of Theorem 1 for $T > T_0 \vee \exp\left\{\dfrac{R_0}{h}\right\}$*

$$\mathsf{P}\left\{\sup_{z\in[0,H]} |\mathrm{B}_T(z,\widehat{\theta}_T) - \mathrm{B}(z)| \geq h(\ln T)T^{-1/2}\right\} \leq AT^{-ah}.$$

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ON SOME INITIAL-BOUNDARY VALUE PROBLEMS FOR PSEUDO-DIFFERENTIAL EQUATIONS RELATED TO A ROTATIONALLY INVARIANT α-STABLE STOCHASTIC PROCESS

MYKHAILO OSYPCHUK, MYKOLA PORTENKO

The following pseudo-differential equation of parabolic type

$$\frac{\partial u}{\partial t} = \mathbf{A}u \tag{1}$$

is considered, where $\mathbf{A}$ is a pseudo-differential operator with its symbol given by the function $(-c|\xi|^{\alpha})_{\xi\in\mathbb{R}^d}$ (parameters $c > 0$ and $\alpha \in (1,2)$ are fixed). It is well-known that the operator $\mathbf{A}$ is the generator of a rotationally invariant α-stable process in $\mathbb{R}^d$ (denote it by $(x(t))_{t\geq 0}$). A fundamental solution of equation (1) (i.e., transition probability density of the process mentioned above) is given by

$$g(t,x,y) = (2\pi)^{-d}\int_{\mathbb{R}^d} \exp\left\{i(\xi, x-y) - ct|\xi|^{\alpha}\right\} d\xi, \tag{2}$$

for $t > 0$, $x \in \mathbb{R}^d$, and $y \in \mathbb{R}^d$. It means that for any continuous bounded function $(\varphi(x))_{x\in\mathbb{R}^d}$, the function

$$u(t,x,\varphi) = \mathbb{E}_x\varphi(x(t)) = \int_{\mathbb{R}^d} g(t,x,y)\varphi(y)\,dy, \quad t>0,\ x\in\mathbb{R}^d, \tag{3}$$

is a solution to equation (1) satisfying the initial condition

$$u(0+,x,\varphi) = \varphi(x), \quad x\in\mathbb{R}^d. \tag{4}$$

In the theory of partial *differential* equations of parabolic type the notion of a single-layer potential is used for solving some initial-boundary value problems [1, Ch. 5]. In our paper [2], we introduced such a notion for equation (1), namely given a smooth closed bounded surface S in $\mathbb{R}^d$ and a continuous function $(\psi(t,x))_{t>0,x\in S}$ satisfying the inequality $|\psi(t,x)| \leq Ct^{-\beta}$ for all $t > 0$ and $x \in S$ with some constants $C > 0$ and $\beta < 1$, we defined the function

$$U(t,x) = \int_0^t d\tau \int_S g(t-\tau,x,y)\psi(\tau,y)\,d\sigma_y, \quad t>0,\ x\in\mathbb{R}^d, \tag{5}$$

where the inner integral was a surface one. This function is called a single-layer potential associated with equation (1). As was proved in [2], it is continuous in $(t,x)\in(0,+\infty)\times\mathbb{R}^d$, satisfies equation (1) in the region $(t,x)\in(0,+\infty)\times\left(\mathbb{R}^d\setminus S\right)$ and possesses the following property (an analogy to the well-known theorem on the jump of the (co-)normal derivative of a single-layer potential in the theory of *differential* equations)

$$\mathbf{B}_{\nu(x)}U(t,\cdot)(x\pm) = \mp\psi(t,x) + \int_0^t d\tau \int_S \mathbf{B}_{\nu(x)}g(t-\tau,\cdot,y)(x)\psi(\tau,y)\,d\sigma_y \tag{6}$$

valid for $t > 0$ and $x \in S$, where $\nu(x)$ is a unit outer normal vector to S at $x \in S$, $\mathbf{B}_{\nu(x)}$ is a pseudo-differential operator whose symbol is given by $(2ci|\xi|^{\alpha-2}(\xi,\nu(x)))_{\xi\in\mathbb{R}^d}$, and $f(x+)$ (resp., $f(x-)$) for $x \in S$ means the limit value of $f(z)$, as z approaches x in a non-tangent way such that $(z, \nu(x)) > 0$ (resp., $(z, \nu(x)) < 0$). The integral in (6) is the so-called direct value of the action of $\mathbf{B}_{\nu(x)}$ on $U(t, x)$.

The main result of our report consists in constructing a fundamental solution to the following problem (see [3]).

Let a pair of continuous functions $(q(x))_{x\in S}$ and $(r(x))_{x\in S}$ (the second one with positive values) be given. For a fixed continuous bounded function $(\varphi(x))_{x\in\mathbb{R}^d}$, a continuous function $(W(t,x))_{t>0,x\in\mathbb{R}^d}$ is being looked for such that it satisfies:

- equation (1) in the region $(t, x) \in (0, +\infty) \times \left(\mathbb{R}^d \setminus S\right)$;
- initial condition (4) for all $x \in \mathbb{R}^d$;
- the following boundary condition ($t > 0$, $x \in S$)

$$\frac{1+q(x)}{2}\mathbf{B}_{\nu(x)}W(t,\cdot)(x+) - \frac{1-q(x)}{2}\mathbf{B}_{\nu(x)}W(t,\cdot)(x-) = r(x)W(t,x).$$

In the case of $q(x) \equiv 0$ the solution of this problem can be written as follows

$$W(t,x) = \mathbb{E}_x\varphi(x(t))e^{-\eta_t}, \quad t > 0,\ x \in \mathbb{R}^d, \tag{7}$$

where η_t is a W-functional of $(x(t))_{t\geq 0}$ defined by its characteristic

$$\mathbb{E}_x\eta_t = \int_0^t d\tau \int_S g(\tau, x, y)r(y)\, d\sigma_y, \quad t > 0,\ x \in \mathbb{R}^d.$$

If the function q does not vanish identically, then the solution of this problem is associated with some pseudo-process in a way analogous to (7).

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SIMULATION OF FRACTIONAL BROWNIAN MOTION WITH GIVEN RELIABILITY AND ACCURACY IN THE SPACE $C([0,T])$

ANATOLII PASHKO, OLGA VASYLYK

We suggest a model and derive conditions for simulation of a fractional Brownian motion with parameter $\alpha \in (0,2)$ with given reliability $1-\delta$, $0<\delta<1$, and accuracy $\varepsilon>0$ in the space $C([0,T])$.

Let (Ω, Σ, P) be a standard probability space and T be a parametric space ($T=[0,T]$ or $T=[0,\infty]$). A random process $\{W_\alpha(t), t\in T\}$ is called fractional Brownian motion with parameter $\alpha\in(0,2)$, if it is a Gaussian process with zero mean $EW_\alpha(t)=0$ and correlation function

$$R(t,s)=\frac{1}{2}\big(|t|^\alpha+|s|^\alpha-|t-s|^\alpha\big),$$

such that $W_\alpha(0)=0$.

A fractional Brownian motion can be represented in the form of a random series [1]. In paper [3], there was constructed a model of a fractional Brownian motion based on such series representation. In [2] we continue our study, presenting a method for simulation of fractional Brownian motion basing on its spectral representation.

In particular, a fractional Brownian motion with parameter $\alpha\in(0,2)$ can be represented in the form of the following stochastic integral [4]:

$$W_\alpha(t)=\frac{A}{\sqrt{\pi}}\bigg(\int_0^\infty\frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\xi(\lambda)-\int_0^\infty\frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\bigg),\ t\in[0,T],$$

where $\xi(\lambda),\eta(\lambda)$ are independent real valued standard Wiener processes with

$$E\xi(\lambda)=E\eta(\lambda)=0,\quad E\big(d\xi(\lambda)\big)^2=E\big(d\eta(\lambda)\big)^2=d\lambda,$$

$$A^2=\left\{\frac{2}{\pi}\int_0^\infty\frac{1-\cos(\lambda)}{\lambda^{\alpha+1}}d\lambda\right\}^{-1}=\left\{-\frac{2}{\pi}\Gamma(-\alpha)\cos\left(\frac{\alpha\pi}{2}\right)\right\}^{-1}.$$

Let us take an interval $[0,\Lambda]$, $\Lambda>0$, and represent the process $W_\alpha=\{W_\alpha(t),t\in[0,T]\}$ in the form

$$W_\alpha(t)=W_\alpha(t,[0,\epsilon])+W_\alpha(t,[\epsilon,\Lambda])+W_\alpha(t,[\Lambda,\infty]),$$

where $0<\epsilon<\Lambda$ and

$$W_\alpha(t,[a,b])=\frac{A}{\sqrt{\pi}}\bigg(\int_a^b\frac{\cos(\lambda t)-1}{\lambda^{\frac{\alpha+1}{2}}}d\xi(\lambda)-\int_a^b\frac{\sin(\lambda t)}{\lambda^{\frac{\alpha+1}{2}}}d\eta(\lambda)\bigg).$$

Let $0=\lambda_0<\lambda_1<...<\lambda_M=\Lambda$ be a partition of the interval $[0,\Lambda]$, such that $\lambda_1=\epsilon$. We construct a *model* of the process W_α in the following way:

$$S_M(t,\Lambda)=\frac{A}{\sqrt{\pi}}\bigg(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda_i^{\frac{\alpha+1}{2}}}(\xi(\lambda_{i+1})-\xi(\lambda_i))-$$

$$-\sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda_i^{\frac{\alpha+1}{2}}}(\eta(\lambda_{i+1})-\eta(\lambda_i))\Bigg) =$$

$$= \frac{A}{\sqrt{\pi}}\left(\sum_{i=1}^{M-1}\frac{\cos(\lambda_i t)-1}{\lambda_i^{\frac{\alpha+1}{2}}}X_i - \sum_{i=1}^{M-1}\frac{\sin(\lambda_i t)}{\lambda_i^{\frac{\alpha+1}{2}}}Y_i\right), \quad t\in[0,T],$$

where $\{X_i, Y_i\}, i = 1, 2, \ldots, M-1$, are independent Gaussian random variables with $EX_i = EY_i = 0$, $EX_i^2 = EY_i^2 = \lambda_{i+1} - \lambda_i$.

Theorem 1. *The model $S_M(t,\Lambda)$ approximates the process W_α with a given reliability $1-\delta$, $0<\delta<1$, and accuracy $\varepsilon>0$ in the space $C([0,T])$ if*

$$\gamma_0 < \varepsilon, \quad \frac{\beta\gamma_0}{K} \le \frac{\varepsilon T^\nu}{2^\nu(\exp\{1/2\}-1)^\nu},$$

$$2\exp\left\{-\frac{(\varepsilon-\gamma_0)^2}{2\gamma_0^2}\right\}\left(\frac{(\varepsilon-\gamma_0)T^b}{2^b\gamma_0(1-b/\nu)}\left(\frac{\varepsilon K}{\beta\gamma_0}\right)^{\frac{b}{\nu}}+1\right)^{\frac{2}{b}} < \delta,$$

where numbers b and ν are such that $0<b<\nu<\frac{\alpha}{2}$, $\beta=\min\{\gamma_0, \frac{K}{2\nu}\}$,

$$\gamma_0 = \frac{A}{\sqrt{\pi}}\left(\frac{T^2\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2}{\alpha\Lambda^\alpha}+\frac{4T^2}{3}\left(1+\left(\frac{\alpha+1}{2}\right)^2\right)\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{\lambda_i^{\alpha+1}}\right)^{1/2},$$

$$\begin{aligned} K &= \frac{A\sqrt{3}}{\sqrt{\pi}}\left[\frac{T^{2-2\nu}\lambda_1^{2-\alpha}}{2-\alpha}+\frac{2^{2-2\nu}}{(\alpha-2\nu)\Lambda^{\alpha-2\nu}}+\right.\\ &+ 2^{4-2\mu}T^{2(\mu-\nu)}\left(\frac{4}{2\mu-\alpha}\sum_{i=1}^{M-1}(\lambda_{i+1}-\lambda_i)^{2\mu-\alpha}+\right.\\ &+ \left.\left.\left(\frac{\alpha+1}{2}\right)^2\sum_{i=1}^{M-1}\frac{(\lambda_{i+1}-\lambda_i)^3}{3\lambda_i^{3-(2\mu-\alpha)}}\right)\right]^{\frac{1}{2}}, \ \mu\in\left(\frac{\alpha}{2};\frac{\alpha+1}{2}\right)\cap(0;1]. \end{aligned}$$

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LIMIT THEOREMS OF SZEGÖ'S TYPE FOR ERGODIC OPERATORS

LEONID PASTUR

We consider an asymptotic setting for ergodic operators in $l^2(\mathbb{Z})$ generalizing that for the Szegö theorem on the asymptotics of determinants of finite-dimensional restrictions of the Toeplitz and discrete convolution operators [5]. The setting is motivated by certain problems of quantum information theory (see. e.g. [1]) and, we believe, is of independent interest. It is formulated via the asymptotic trace formula determined by a triple consisting of an ergodic operator H and two functions, the *symbol* a and the *test function* φ. In the frameworks of this setting we analyze two important cases in which H is the discrete Schrodinger operator with *random i.i.d.* potential and the same operator with *quasiperiodic* potential. In the random case we find that for smooth symbols and test functions the corresponding asymptotic formula contains a new subleading term, which is random and proportional to the square root of the length of the interval of restriction. The origin of the term is the Gaussian fluctuations of the corresponding trace, i.e, in fact, a certain Central Limit Theorem in the spectral context. We also present an example of a discontinuous symbol for which the subleading term is bounded, being the sum of two ergodic processes bounded with probability 1, while for the Toeplitz discrete convolution operators and the same symbol the subleading term grows logarithmically in the length of the interval of restriction. In the quasiperiodic case and for smooth symbols the subleading term is bounded as in the Szegö theorem but unlike the theorem, where the term does not depend on the length of the interval of restriction, in the quasiperiodic case the term is the sum of two quasiperiodic functions in the length.

The talk is based on the works [2, 3, 4].

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REGULARLY LOG-PERIODIC FUNCTIONS

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Regularly log-periodic function (RLP) is a function f of the form

$$f(x) = x^{\rho}\ell(x)H(\ln x), \quad x \geq A,$$

where $A > 0$, $\rho \in \mathbf{R}$, H is a positive continuous periodic function and ℓ is a slowly varying function, i.e. ℓ is measurable and for all $\lambda > 0$

$$\lim_{x\to\infty} \frac{f(\lambda x)}{f(x)} = 1.$$

These functions naturally generalize Karamata regularly varying (RV) functions.

The notion "regularly log-periodic" appears in [2], but such functions arise in different problems of probability theory and mathematical analysis. Some problems where RLP functions emerge and can be of practical use will be considered in this talk.

Besides, the generalization of Karamata theorem on the asymptotic behavior of integrals from RV function on the class RLP will be presented. This result is an integral criterion for a function to belong to the class RLP.

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ON PERTURBATIONS OF ORDINARY DIFFERENTIAL EQUATIONS WITH NON-LIPSHITZ COEFFICIENTS BY A SMALL-NOISE

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We study the limit behavior of the sequence

$$Y^{\varepsilon}(t)=\int_0^t a(Y^{\varepsilon}(s))ds+\varepsilon B_{\alpha}(t),\ t\geq 0, \tag{1}$$

as $\varepsilon\to 0$, where B_α is an α-stable process, $\alpha\in(1,2]$, and $a(y)\sim c_{\pm}\mathrm{sgn}(y)|y|^{\beta}$ as $y\to 0\pm$, $\beta<1$, $c_{\pm}>0$.

Note that the formal limit equation

$$Y^{0}(t)=\int_0^t a(Y^{0}(s))ds \tag{2}$$

has a non-unique solution. So the limit in (1) can be considered as a natural selection of a solution in (2).

It appears that the limit of $\{Y^{\varepsilon}\}$ as $\varepsilon\to 0$ is closely related with a long time behavior of a solution to

$$Y(t)=\int_0^t (c_{+}1_{Y(s)>0}-c_{-}1_{Y(s)<0})|Y(s)|^{\beta}ds+B_{\alpha}(t),\ t\geq 0,$$

as $t\to\infty$.

The asymptotic behavior of stochastic differential equations growth was considered in [1] when the noise is Wiener, see also [2, 3].

We also consider a multidimensional generalization of (1), where the vector field $a:\mathbb{R}^d\to\mathbb{R}^d$ is locally Lipschitz everywhere except of a hyperplane $\mathbb{R}^{d-1}\times\{0\}$. The corresponding limit depends on the normal component of the drift at the upper and lower half-spaces in a neighborhood of the hyperplane.

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GENERALIZED DERIVATIVE SAMPLING SERIES

TIBOR K. POGÁNY

Master generalized sampling series expansion is presented for entire functions (signals) coming from a class which members satisfy an extended exponential boundedness condition. Firstly, estimates are given for the remainder of Maclaurin series of those functions and consequent derivative sampling results are derived and discussed. These results are employed in evaluating the related remainder term of signals which occur in sampling series expansion of stochastic processes and random fields (not necessarily stationary or homogeneous) which spectral kernel satisfy the relaxed exponential boundedness. The derived truncation error upper bounds enable to obtain mean-square master generalized derivative sampling series expansion formulae either for harmonizable Piranashvili-type stochastic processes or for random fields. Finally, being the sampling series convergence rate exponential, almost sure $\mathbb{P}$ sampling series convergence rate is established.

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RISK ESTIMATION OF A CARRINGTON-LIKE GEOMAGNETIC STORM

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A geomagnetic storm is a disturbance in the magnetosphere quantified by changes in the Dst (disturbance-storm time) index. This index measures the globally averaged change of the horizontal component of the Earth's magnetic field at the magnetic equator and it is recorded once per hour. During quiescent times, the Dst index varies between -20 and +20 nT (nanotesla). The Carrington event is the largest known example of geomagnetic storm, occurred by the end of August and early September 1859 and associated to a minimum Dst under -850 nT. Richard C. Carrington was observing sunspots on the solar disk and saw a large solar flare with optical brightness lasting several minutes and equaling that of the background sun, due to the destabilization of a large region of the sun causing an extremely fast coronal mass ejection towards Earth. Nowadays, a Carrington-like geomagnetic storm would be catastrophic for electrical systems and communications. The Dst index has been traditionally modelled by means of its physical properties [1, 2], although some work has also focused on exploring its statistical properties [3]. As far as we know, all efforts in statistical modelling have been based on the assumption that the occurrence of a geomagnetic storm follows an homogeneous Poisson counting process (see for instance [1]).

To analyse the process of temporal occurrence of geomagnetic storms we use the Dst index, recorded hourly from 1957-01-01 to 2017-12-31 and available from the World Data Center for Geomagnetism in Kyoto. When the Dst signal crosses a fixed negative threshold from above this defines the occurrence time or starting time of a geomagnetic storm with an intensity limited by the threshold. The time between two consecutive storms below the threshold is just the difference of their occurrence times. We have found that the distributions of inter-occurrence times seem to be well fitted by Weibull distributions. In terms of the complementary cumulative distribution function, the Weibull distribution takes the form $S(t) = P(X > t) = e^{(-t/\tau)^\gamma}$, where X is the random variable representing inter-occurrence times and γ, τ are respectively the parameters of shape and scale.

It is found that the scale parameter of the inter-occurrence times distribution grows exponentially with the absolute value of the intensity threshold defining the storm, whereas the shape parameter keeps rather constant (see Figure 1).

Therefore, the inter-occurrence times were fitted using a Weibull regression model where the scale parameter changes with the threshold of the storm, T, according to $\log(\tau) = \beta_0 + \beta_1 T$ and the shape parameter γ is constant. The estimates are $\log(\hat{\gamma}) = -0.39$ (SE $= 0.023$), $\beta_0 = 2.96$ (SE $= 0.17$) and $\beta_1 = -0.0121$ (SE $= 0.0008$).

Knowing that the original Carrington event happened in 1859, about 58000 days ago, one can compute the probability of having a Carrington or more intense event during the next decade (2018-2027) conditioned to the fact that no event like this has happened since 1859,

$$P(X \leq t_C + t_d \mid X \geq t_C) = \frac{S(t_C) - S(t_C + t_d)}{S(t_C)} = 1 - \exp\left[\left(\frac{t_C}{\tau}\right)^\gamma - \left(\frac{t_C + t_d}{\tau}\right)^\gamma\right] = 0.0092,$$

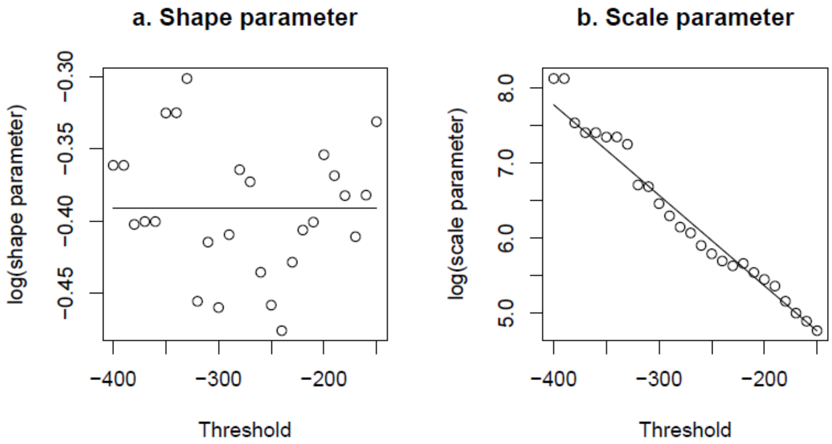

FIGURE 1. Relationship between Dst threshold (in nT) and Weibull shape (a.) and scale (b.) parameters, in log-scale, with scale parameter in days. Intensity thresholds range from -400 nT to -150 nT. The points correspond to maximum-likelihood estimates of the shape and scale parameters for fixed threshold values.

with $t_C = 58000$ days and $t_d = 3652$ days (10 years). According to this model, the estimated probability is 0.92%, with a 95% confidence interval equal to $[0.46\%, 1.88\%]$. The value reported in [1] was about 12%, in sharp contrast with our result.

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CONSTRUCTIVE STOCHASTIC INTEGRAL REPRESENTATION OF WIENER FUNCTIONAL

OMAR PURTUKHIA

In the theory of stochastic processes, the representation of functionals of Wiener process by stochastic integrals, also known as martingale representation theorem, asserts that a functional that is measurable with respect to the filtration generated by Wiener process can be written in terms of the Ito stochastic integral with respect to this Wiener process. The theorem only asserts the existence of the representation and does not help to find it explicitly. In some cases it is possible to define the form of a representation using the Malliavin calculus if the functional is Malliavin differentiable. Sufficiently well-behaved Frechet-differentiable functionals have an explicit representation as a stochastic integral in which the integrand has the form of conditional expectations of the differential. We consider non-smooth (in the sense of Malliavin) functionals and suggest some methods for obtaining constructive martingale representation theorems. The obtained results can be used to establish the existence of a hedging strategy in various European options with appropriate payoff functions.

The first proof of the martingale representation theorem was implicitly provided by Ito (1951) himself. Many years later, Dellacherie (1974) gave a simple new proof of Ito's theorem using Hilbert space techniques. One of the pioneer work on explicit descriptions of the integrand is certainly the one by Clark (1971), according to which if F is a square integrable $\Im_T^W$-measurable random variable, then there exist a square integrable $\Im_t^W$-adapted random process $\varphi(t,\omega)$ such that

$$F = EF + \int_0^T \varphi(t,\omega)dW_t(\omega).$$

In general, the finding of explicit expression for integrand $\varphi(t,\omega)$ of stochastic integral is very difficult problem. According to Ocone (1984) $\varphi(t,\omega) = E[D_tF|\Im_t^W]$ (so called Clark-Ocone formula), where D_t is the so called Malliavin stochastic derivative. A different method for finding the process $\varphi(t,\omega)$ was proposed by Shiryaev, Yor and Graversen (2003, 2006), which was based on the Ito (generalized) formula and the Levy theorem for the Levy martingale $M_t = E[F|\Im_t^W]$ connected with the considered functional F. Later on, using the Clark-Ocone formula, Renaud and Remillard (2006) have established explicit martingale representations for path-dependent Wiener functionals.

In all cases described above investigated functionals, were stochastically (in Malliavin sense) smooth. We study the problem of stochastic integral representation of stochastically nonsmooth functional. In [1], we also considered the stochastically nonsmooth path-dependent Wiener functional.

Theorem 1. [Glonti, Purtukhia, 2014] For the functional $F = (W_T - C_1)^+ I_{\{W_T^* \le C_2\}}$ ($C_2 \ge C_1 > 0$, $W_T^* = \sup_{t\in[0,T]} W_t$) the following stochastic integral representation is fulfilled

$$F = EF - \int\limits_0^T \frac{2(C_2 - C_1)}{\sqrt{T-s}} \varphi(\frac{C_2 - W_t}{\sqrt{T-t}}) dW_t +$$

$$+ \int\limits_0^T \{\Phi(\frac{W_t - C_1}{\sqrt{T-t}}) - \Phi(\frac{W_t - 2(C_2 - C_1)}{\sqrt{T-t}})\} dW_t.$$

It turned out that the requirement of smoothness of the functional can be weakened (see, [2]). In particular, we generalized the Clark-Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method for finding of the integrand.

Theorem 2. [Glonti, Purtukhia, 2017] Suppose that $g_t = E[F|\Im_t^W]$ is Malliavin differentiable ($g_t(\cdot) \in D_{2,1}^W$) for almost all $t \in [0, T)$. Then we have the stochastic integral representation

$$g_T = F = EF + \int_0^T \nu_u dW_u \quad (P - a.s.),$$

where

$$\nu_u := \lim_{t \uparrow T} E[D_u g_t | \Im_u^W] \quad in \;\; the \quad L_2([0,T] \times \Omega).$$

It is clear that there are also such functionals which don't satisfy even the weakened conditions, i.e. the nonsmooth functionals whose conditional mathematical expectations is not stochastically differentiable too (see, for example,[3]). We will consider the typical representative of such functionals, in particular, the Wiener functional of integral type $F = \int_0^T f(W_t)dt$. We denote by $V(t,x) := E[\int_t^T f(W_s)ds | B_t = x]$.

Theorem 3. If the deterministic function $V(t,x)$ satisfies the requirements of the generalized Ito theorem, then the following stochastic integral representation is fulfilled

$$\int_0^T f(W_t)dt = E[\int_0^T f(W_t)dt] + \int_0^T \partial/\partial x V(t, W_t) dW_t.$$

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THE ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS OF EQUATIONS DRIVEN BY GENERAL STOCHASTIC MEASURES

VADYM RADCHENKO

Let $\mathsf{L}_0 = \mathsf{L}_0(\Omega, \mathcal{F}, \mathsf{P})$ be the set of all real-valued random variables defined on the complete probability space $(\Omega, \mathcal{F}, \mathsf{P})$ (more precisely, the set of equivalence classes). Convergence in L_0 means the convergence in probability. Let X be an arbitrary set and $\mathcal{B}$ a σ-algebra of subsets of X.

Definition 1. A σ-additive mapping $\mu:\ \mathcal{B} \to \mathsf{L}_0$ is called *stochastic measure* (SM).

Assumption A1. μ is a SM on Borel subsets of $[0,T]$, and the process $\mu_t = \mu((0,t])$ has continuous paths on $[0,T]$.

Assumption A2. There exists a real-valued finite measure m on $(\mathsf{X}, \mathcal{B})$ with the following property: if a measurable function $h : \mathsf{X} \to \mathbb{R}$ is such that $\int_{\mathsf{X}} h^2 \,\mathrm{d}\mathsf{m} < +\infty$ then h is integrable with respect to μ on X.

We do not assume the moment existence or martingale properties for SM.

The following symmetric integral of random functions with respect to stochastic measures was considered in [1].

Definition 2. Let ξ_t and η_t be random processes on $[0,T]$, $0 = t_0^n < t_1^n < \cdots < t_{j_n}^n = T$ be a sequence of partitions such that $\max_k |t_k^n - t_{k-1}^n| \to 0$, $n \to \infty$. We define

$$\int_{(0,T]} \xi_t \circ \mathrm{d}\eta_t := \mathrm{p}\lim_{n\to\infty} \sum_{k=1}^{j_n} \frac{\xi_{t_{k-1}^n} + \xi_{t_k^n}}{2} \left(\eta_{t_k^n} - \eta_{t_{k-1}^n}\right) \tag{1}$$

provided that this limit in probability exists.

Assumption A3. V_t is a continuous process of bounded variation on $[0,T]$.

Theorem 1 ([1])**.** *Let A1 and A3 hold, $f \in \mathbb{C}^{1,1}(\mathbb{R}^2)$. Then integral* (1) *of $f(\mu_t, V_t)$ with respect to μ_t is well defined, and*

$$\int_{(0,T]} f(\mu_t, V_t) \circ \mathrm{d}\mu_t = G(\mu_t, V_t) - G(\mu_0, V_0) - \int_{(0,T]} G_2'(\mu_t, V_t)\,\mathrm{d}V_t,$$

where $G(x,v) = \int_0^x f(y,v)\,\mathrm{d}y$.

We will consider a stochastic equation of the form

$$\circ\,\mathrm{d}X_t = \sigma(X_t) \circ \mathrm{d}\mu_t + b(X_t, t)\,\mathrm{d}t, \quad 0 \le t \le T. \tag{2}$$

Definition 3. A process X_t, $0 \le t \le T$ is a *solution* to (2) if:

1) $X_t = f(\mu_t, Y_t)$, $f \in \mathbb{C}^{2,1}(\mathbb{R}^2)$, Y_t is a continuous process of bounded variation;

2) for any process $Z_s = \psi(\mu_s, X_s)$, $\psi \in \mathbb{C}^{1,1}(\mathbb{R}^2)$, we have

$$\int_{(0,t]} Z_s \circ \mathrm{d}X_s = \int_{(0,t]} Z_s \sigma(X_s) \circ \mathrm{d}\mu_s + \int_{(0,t]} Z_s b(X_s, s)\,\mathrm{d}s, \quad t \in [0,T].$$

Assumption A4. 1) $\sigma \in \mathbb{C}^2(\mathbb{R})$ and the derivatives σ', σ'' are bounded;
2) $b \in \mathbb{C}(\mathbb{R}^2)$;
3) for each $c > 0$ there exists a $L(c)$ such that

$$|b(x,t) - b(y,t)| \le L(c)|x - y|, \quad |x|,\ |y| \le c;$$

4) b is bounded.
Let $F : \mathbb{R}^2 \to \mathbb{R}$ be the solution of the equation

$$\frac{\partial F}{\partial r}(r,x) = \sigma(F(r,x)), \quad F(0,x) = x,$$

which exists globally because of our assumptions. Set $H(r,x) = F^{-1}(r,x)$, where the inverse is taken with respect to x.

Theorem 2 ([1]). *Let A2 and A4 hold, X_0 be an arbitrary random variable. Then equation* (2) *has a unique solution $X_t = F(\mu_t, Y_t)$, where Y_t is the solution of the random equation*

$$Y_t = H(0, X_0) + \int_0^t \frac{\partial H}{\partial x}(\mu_s, F(\mu_s, Y_s)) b(F(\mu_s, Y_s), s)\, \mathrm{d}s.$$

We will consider the averaging principle for this equations. For each $\varepsilon > 0$ consider the equation

$$\circ\, \mathrm{d}X_t^\varepsilon = \sigma(X_t^\varepsilon) \circ \mathrm{d}\mu_t + b(X_t^\varepsilon, t/\varepsilon)\, \mathrm{d}t, \quad 0 \le t \le T. \tag{3}$$

and its averaged form

$$\circ\, \mathrm{d}\bar{X}_t = \sigma(\bar{X}_t) \circ \mathrm{d}\mu_t + \bar{b}(\bar{X}_t)\, \mathrm{d}t, \quad 0 \le t \le T. \tag{4}$$

Assumption A5. Function $G(y,r) = \int_0^r (b(y,s) - \bar{b}(y))\, \mathrm{d}s$, $r \in \mathbb{R}_+,\ y \in \mathbb{R}$ is bounded.

Theorem 3. *1) Assume that A1, A4, and A5 hold, X_t^ε and $\bar{X}_t$ are the solutions of* (3) *and* (4) *respectively. Then for each $\omega \in \Omega$*

$$\sup_{t \in [0,T]} |X_t^\varepsilon - \bar{X}_t| \to 0, \quad \varepsilon \to 0.$$

2) Let, in addition, A2 holds. Then the set of the random variables

$$\frac{\sup_{t \in [0,T]} |X_t^\varepsilon - \bar{X}_t|}{\varepsilon^{1/3}}, \quad \varepsilon > 0$$

is bounded in probability.

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BROWNIAN TRADING EXCURSIONS

THORSTEN RHEINLANDER

We study a parsimonious but non-trivial model of the latent limit order book where orders get placed with a fixed displacement from an efficient price process, i.e.some process in-between best bid and best ask, and get executed whenever this efficient price reaches their level. This mechanism corresponds to the fundamental solution of the stochastic heat equation with multiplicative noise for the relative order volume distribution. We classify various types of trades, and introduce the trading excursion process which is a Poisson point process. This allows to derive the Laplace transforms of the times to various trading events under the corresponding intensity measure.

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OPTIMAL PACKING OF BROWNIAN BALLS

SYLVIE ROELLY

We consider a system of n equal hard balls in Euclidean space, undergoing Brownian dynamics and interacting via a mutual attraction force: for $i \in \{1, \cdots, n\}$, denoting by $X_i(t)$ the position of the center of the ith ball at time t,

$$dX_i(t) = dB_i(t) - a\sum_{j=1}^{n}(X_i(t) - X_j(t))dt + \sum_{j=1}^{n}(X_i(t) - X_j(t))\, dL_{ij}(t)$$

where the random processes L_{ij} are called *collision local times*.

We prove that such Langevin stochastic evolution converges asymptotically in time towards an equilibrium state. When the attraction parameter a is large, this stationary measure is asymptotically connected with the famous geometry problem of close-packing of equal spheres.

The case of infinitely many balls will also be discussed.

These results were obtained via collaborations with Patrick Cattiaux (Toulouse), Myriam Fradon (Lille) and Alexei M. Kulik (Kyiv).

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GENERALIZED COUPLINGS

MICHAEL SCHEUTZOW

The *coupling* technique has been successfully applied to show uniqueness of an invariant probability measure π of a Markov processes and convergence of transition probabilities to π. The first result in this direction was published by Wolfgang Doblin in 1940. By definition, a coupling of two probability measures μ and ν on the same measurable space $(E, \mathcal{E})$ is any probability measure ξ on the product space $(E \times E, \mathcal{E} \otimes \mathcal{E})$ with respective marginals μ and ν. In this talk we introduce *generalized couplings* which, by definition, are probability measures ξ on the product space for which the marginals are not necessarily equal to but only absolutely continuous with respect to μ and ν respectively. In some applications it is more appropriate to replace the word *absolutely continuous* by *close to* (in an appropriate sense).

We will give several applications of this concept. In particular we will see, how one can show uniqueness of a weak solution of a stochastic delay differential equation driven by Brownian motion in the case where the coefficients are merely Hölder continuous and the noise is non-degenerate. We will also show how to obtain exponential convergence rates of the transition probabilities to the invariant probability measure assuming that a suitable Lyapunov function exists.

This is joint work with Alexey Kulik (Kiev) and Oleg Butkovsky (Berlin and Haifa)

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PESIN'S FORMULA FOR ISOTROPIC BROWNIAN FLOWS

VITALII SENIN

Pesin's formula is a relation between the entropy of a dynamical system and its positive Lyapunov exponents. This formula was first established by Pesin in the late 1970s for some deterministic dynamical systems acting on a compact Riemannian manifold. Later on the same formula was obtained in some other settings. For example, different authors have proved the formula for some random dynamical systems, or have relaxed the condition of state space compactness. Nevertheless, it has never been obtained for dynamical systems with invariant measure, which is infinite. The problem is that if invariant measure is infinite, then the notion of entropy becomes senseless. Invariant measure of isotropic Brownian flows is the Lebesgue measure on $\mathbb{R}^d$, which is, clearly, infinite. Nevertheless, we define entropy for such a kind of flows using their translation invariance. For the definition we exploit ideas of Brin and Katok, see [1]. Then we study the analogue of Pesin's formula for these flows using the defined entropy.

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BRANCHING RANDOM WALKS

ZHAN SHI

Branching random walks and branching Brownian motions are branching systems where each individual, also referred to as particle, is associated with a spatial parameter representing the fitness value of the individual. They are connected to several other important topics in mathematics, computer science, physics and biology. I am going to give an elementary and self-contained introduction to the study of the structure of extreme positions in branching random walks and branching Brownian motions.

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CONSISTENT TLS ESTIMATOR IN HETEROSCEDASTIC LINEAR ERROR-IN-VARIABLES MODEL

SERGIY SHKLYAR

Consider linear regression with errors in variables. Let the true regressors be nonrandom variables that make a vector $\xi_i \in \mathbb{R}^p$ on each observation. The true response variable is $\eta_i = \beta^\top \xi_i$. The regressors and response variables are observed with errors:

$$\begin{cases} x_i = \xi_i + \delta_i, \\ y_i = \beta^\top \xi_i + \epsilon_i. \end{cases}$$

Assume that the augmented vectors of errors $\vec{\epsilon}_i = (\delta_i^\top, \epsilon_i)^\top$ are mutually independent; they have zero means and known covariance matrices Σ_i. The matrices Σ_i are assumed to be known, but may be different for different observations. The points (x_i, y_i), $i = 1, \ldots, m$, are observed and the parameter β is estimated.

Consider two estimators of the parameter β. The element-wise weighted total least squares (EW-TLS) estimator is defined in Kukush and Van Huffel. When the matrices Σ_i are nonsingular, the EW-TLS minimizes the functional

$$Q(\beta) = \sum_{i=1}^{m} \min_{t \in \mathbb{R}^p} (x_i^\top - t^\top, y_i - \beta^\top t) \Sigma_i^{-1} \begin{pmatrix} x_i - t \\ y_i - \beta^\top t \end{pmatrix}.$$

For the other estimator, the matrices Σ_i were preliminarily averaged over the observations.

The consistency conditions for the TLS-EW estimator are relaxed and for the TLS estimator with averaging covariance matrix are provided.

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ESTIMATE OF LIMIT SEMI-CONTINUOUS KILLED MARKOV DECISION PROCESS

PAVLO SHPAK, YAROSLAV YELEYKO

We consider a sequence of semi-continuous killed Markov decision processes [1] $(X, A, j, p, q, r, c, \mu) \equiv Z_\mu$ which satisfies the following conditions:

(1) set of states $X = \bigcup\limits_{t=m}^{n} X_t$ is a separable metric space;
(2) set of all pairs xx $(x \in X_t)$ belongs $\sigma(X_t \times X_t)$;
(3) subset of killed states $X^* \subset X$ is measurable
(4) $X_m, X_{m+1}, \ldots, X_n$ are disjoint subsets of X;
(5) sets $X_m \cap X^*,\ X_m \setminus X^*,\ X_{m+1} \cap X^*,\ X_{m+1} \setminus X^*, \ldots, X_n \cap X^*,\ X_n \setminus X^*$ are closed subsets of X;
(6) set of controls $A = \bigcup\limits_{t=m+1}^{n} A_t$ is a separable metric space
(7) $A_{m+1}, A_{m+2}, \ldots, A_n$ are disjoint closed subsets of A;
(8) correspondence $A(x)$ is quasicontinuous (if $x_k \to x \in X$ and $a_k \in A(x_k)$, then $\{a_k\}$ has a limit point belonging to $A(x)$);
(9) if $f \in \pounds(X_t)$ [1] and $g(a) = \int\limits_{X_t} p(dx|a) f(x)$ $(a \in A_t)$, then $g \in \pounds(A_t)$;
(10) $j : A \to X$ are correspondence to projection $j(A_{t+1}) = X_t$;
(11) $p(\cdot|a) \equiv \mathbb{P}(x_t = x | a_t = ax_{t-1})$ are probability distributions on X_t;
(12) $q : A \to \mathbb{R}$ is a function on the set of controls (current cost);
(13) $r : X_n \to \mathbb{R}$ is a function on the set of final states (final cost);
(14) $c : X_t \cap X^* \to \mathbb{R}$ is a function on the set of killed states:

$$c(x) \le -\sum_{t=m+1}^{n} \sup_{a_t \in A_t} q(a),\ x \in X_t \cap X^*;$$

And analyze the sufficient conditions of existence of the estimate [2] of limit process. As a result we have a following theorem:

Theorem 1. *Let $Z^k \equiv (X, A, j, p, q^k, r^k, c^k)$ be a sequence of semi-continuous killed Markov decision processes, q^k, c^k, r^k are bounded sequences of functions and*

$$q^k \xrightarrow[k\to\infty]{} q^* \in \pounds(A_t)$$

$$c^k \xrightarrow[k\to\infty]{} c^* \in \pounds(X_t \cap X^*)$$

$$r^k \xrightarrow[k\to\infty]{} r^* \in \pounds(X_n)$$

Then estimate of the limit process $Z^* \equiv (X, A, j, p, q^*, r^*, c^*)$ is equal to the limit of estimates of Z^k:

$$\forall \mu : \ \upsilon^k(\mu) \xrightarrow[k\to\infty]{} \upsilon^*(\mu)$$

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APPLICATIONS OF STOCHASTIC PROCESSES THEORY TO PROBLEMS OF MATHEMATICAL PHYSICS WITH RANDOM FACTORS

ANNA SLYVKA-TYLYSHCHAK

The paper is devoted to an example of application of estimates of the distribution supremum at infinity for fields from space $Sub_{\varphi}(\Omega)$ (see [1]) to the solution of a hyperbolic type equation of mathematical physics, where $t \in [0, +\infty)$.

Consider the boundary-value problem of the first kind for a homogeneous hyperbolic equation [2]. The problem is whether one can find a function $u = (u\,(x, y)\,,\ x \in [0, \pi]\,,$ $t \in [0, t])$ satisfying the following conditions:

$$\frac{\partial}{\partial x}\left(p(x)\frac{\partial u}{\partial x}\right) - q(x)u - \rho(x)\frac{\partial^2 u}{\partial t^2} = 0;$$

$$x \in [0, \pi]\,,\ t \in [0, +\infty]\,;$$

$$u(0, t) = u(\pi, t) = 0,\ t \in [0, +\infty]\,;$$

$$u(x, 0) = \xi(x),\ \frac{\partial u(x, 0)}{\partial t} = \eta(x),\ x \in [0, \pi]\,.$$

Assume also that $(\xi(x),\ x \in [0, \pi])$ and $(\eta(x),\ x \in [0, \pi])$ are $SSub_{\varphi}(\Omega)$ stochastic processes defined on a common complete probability space $(\Omega,\ \Im,\ P)$, where $\varphi(x) = \frac{|x|^p}{p}$, $|x| > 1$, $p > 1$.

Independently of whether the initial conditions are deterministic or random the Fourier method consists in looking for a solution to the series

$$u\,(x, t) = \sum_{k=1}^{\infty} X_k(x)\left[A_k cos\sqrt{\lambda_k}t + \frac{B_k}{\sqrt{\lambda_k}} sin\sqrt{\lambda_k}t\right],$$

$$x \in [0, \pi]\,,\ t \in [0, +\infty]\,;$$

where

$$A_k = \int_0^{\pi} \xi(x)X_k(x)\rho(x)dx,\ k \geq 1,$$

$$B_k = \int_0^{\pi} \eta(x)X_k(x)\rho(x)dx,\ k \geq 1,$$

and where $\lambda_k,\ k \geq 1$ and $X_k = (X_k(x),\ x \in [0, \pi]),\ k \geq 1$ are eigenvalues and the corresponding orthonormal, with weight $\rho(\cdot)$, eigenfunctions of the following Sturm-Liouville problem

$$\frac{d}{dx}\left(p(x)\frac{dX(x)}{dx}\right) - q(x)X(x) + \lambda\rho(x)X(x) = 0,$$

$$X(0) = X(\pi) = 0.$$

Theorem 1. *Let* $\{u(x,t),\ (x,t)\in V\}$, $V=[0;\pi]\times[0,+\infty)$ *be a separable random field belonging to* $Sub_\varphi(\Omega)$*, where* $\varphi(x)=\frac{|x|^p}{p}$ *for* $|x|>1$, $p>1$*. Assume the following conditions are satisfied.*

(1) $[b_k,b_{k+1}]$, $k=0,1,\ldots$ *is a family of such segments, that* $-\infty<b_k<b_{k+1}<+\infty$, $k=0,1,\ldots$ $V_k=[0;\pi]\times[b_k,b_{k+1}]$, $\bigcup\limits_k V_k=V$.

(2) *Let there exist constants* $a_k>0$ *and* $d>1$*, such that* $\pi>\frac{2}{d}$, $\frac{b_{k+1}-b_k}{2}>\frac{1}{d}$ *and*

$$\sup_{\substack{|x-x_1|\le h,\\ |t-t_1|\le h\\ (x,t),(x_1,t_1)\in V_k}}\tau_\varphi\left(u(x,t)-u(x_1,t_1)\right)\le\frac{a_k}{\left|\ln\left(\frac{1}{|h|}+d\right)\right|^\alpha},$$

for some $|h|$ *and* $\alpha>1-\frac{1}{p}$.

(3) $c=\{c(t),\ t\in R\}$ *is some continuous function, such that* $c(t)>0$, $t\in R$, $c_k=\min\limits_{t\in[t_k,t_{k+1}]}c(t)$.

(4) $\sup\limits_k\frac{1}{c_k}<\infty$, $\sup\limits_k\frac{\ln\left(\pi\cdot\frac{b_{k+1}-b_k}{2}\right)^{\frac{1}{q}}}{c_k}<\infty$.

(5) *The series* $\sum\limits_{k=0}^{\infty}\exp\left\{-\frac{1}{q}\left(\frac{sc_k(1-\theta)}{2\tilde{\varepsilon}_0}\right)^{\frac{1}{q}}\right\}$ *converges for some* s*, such that,* $\sup\limits_k\frac{4\varepsilon_k}{c_k(1-\theta)}<s<\frac{v}{2}$*, where* $\tilde{\varepsilon}_0=\sup\limits_{(x,t)\in V_k}\left(E\left(u(x,t)\right)^2\right)^{\frac{1}{2}}$, $k=0,1,\ldots$

Then

$$P\left\{\sup_{(x,t)\in V}\frac{|u(x,t)|}{c(t)}>v\right\}\le 2\exp\left\{-\frac{1}{q}\left(\frac{v}{s}\right)^{\frac{1}{q}}\right\}\cdot\sum_{k=0}^{\infty}\exp\left\{-\frac{1}{q}\left(\frac{sc_k(1-\theta)}{2\tilde{\varepsilon}_0}\right)^{\frac{1}{q}}\right\}.$$

for $v>\sup\limits_k\frac{\frac{1}{p^{\frac{1}{p}}}\left(2^{\frac{1}{q}}(a)^{\frac{1}{\alpha q}}\frac{(\theta\tilde{\varepsilon}_0)^{1-\frac{1}{\alpha q}}}{1-\frac{1}{\alpha q}}+\theta\tilde{\varepsilon}_0\ln\left(\pi\cdot\frac{b_{k+1}-b_k}{2}\right)^{\frac{1}{q}}\right)}{c_k}\cdot\frac{4}{\theta(1-\theta)}$, $0<\theta<1$.

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CAUCHY PROBLEMS AND INVARIANT MEASURES FOR PARTIAL STOCHASTIC FUNCTIONAL-DIFFERENTIAL EQUATIONS

A. N. STANZHYTSKYI, A. O. TSUKANOVA

We deal with the Cauchy problem for stochastic functional-differential equation

$$du(t,x) = \Big(\Delta_x u(t,x) + f(u_t(x))\Big)dt + \sigma(u_t(x))d\beta(t),\ 0 < t \le T,\ x \in \mathbb{R}^d, \tag{1}$$

$$u(t,x) = \phi(t,x),\ -r \le t \le 0,\ x \in \mathbb{R}^d,\ r > 0, \tag{1*}$$

where $T > 0$ is fixed, $\Delta_x \equiv \sum\limits_{i=1}^{d} \frac{\partial^2}{\partial x_i^2}$ is d-measurable Laplacian in the space variables, W is a Q-Wiener process, f and σ are some given functionals to be specified later, $\phi\colon [-r,0]\times \times\mathbb{R}^d \to \mathbb{R}$ is an initial-datum function, $u_t(x) = u(t+\theta, x)$, $0 \le t \le T$, $x \in \mathbb{R}^d$, $-r \le \theta \le 0$.

Let $(\Omega, \mathcal{F}, \mathbf{P})$ note a complete probability space. Henceforth $L_2^{\rho}(\mathbb{R}^d)$ will denote real Hilbert space with the norm $\|g\|_{L_2^{\rho}(\mathbb{R}^d)} = \left(\int\limits_{\mathbb{R}^d} g^2(x)\rho(x)dx\right)^{\frac{1}{2}}$, where $\rho \in L_1(\mathbb{R}^d)$ is a so-called admissible weight. Let $\{e_n(x), n \in \{1,2,\dots\}\}$ be an orthonormal basis on $L_2(\mathbb{R}^d)$ such that $\sup\limits_{n\in\{1,2,\dots\}} ess\sup\limits_{x\in\mathbb{R}^d} |e_n(x)| \le 1$. We now define $L_2(\mathbb{R}^d)$-valued Q-Wiener process $W(t,x) = W(t,\cdot)$, $t \ge 0$, $x \in \mathbb{R}^d$, as follows $W(t,\cdot) = \sum\limits_{n=1}^{\infty} \sqrt{\lambda_n} e_n(\cdot)\beta_n(t)$, $t \ge 0$, where $\{\beta_n(t), n \in \{1,2,\dots\}\} \subset \mathbb{R}$ are independent standard real-valued one-dimensional Brownian motions on $t \ge 0$, $\{\lambda_n, n \in \{1,2,\dots\}\}$ is a sequence of positive numbers such that $\sum\limits_{n=1}^{\infty} \lambda_n < \infty$. Let $\{\mathcal{F}_t, t \ge 0\}$ be a normal filtration on $\mathcal{F}$. We assume that $W(t,\cdot)$, $t \ge 0$, is a Q-Wiener process with respect to a filtration $\{\mathcal{F}_t, t \ge 0\}$, i.e.,

- $W(t,\cdot)$, $t \ge 0$, is $\mathcal{F}_t$-measurable;
- the increments $W(t+h,\cdot) - W(t,\cdot)$ are independent of $\mathcal{F}_t$ for all $h > 0$ and $t \ge 0$.

Let $L_2([-r;0]; L_2^{\rho}(\mathbb{R}^d))$ will note real Hilbert space with the norm $\|g\|_{L_2([-r;0];L_2^{\rho}(\mathbb{R}^d))} = \left(\int\limits_{-r}^{0}\int\limits_{\mathbb{R}^d} g^2(\theta,x)\rho(x)dxd\theta\right)^{\frac{1}{2}}$. For all $t \ge 0$ u_t is $L_2([-r;0]; L_2^{\rho}(\mathbb{R}^d))$-valued process. Let denote by $H = L_2^{\rho}(\mathbb{R}^d) \times L_2([-r;0]; L_2^{\rho}(\mathbb{R}^d))$ Hilbert space of vectors $U(t,x) = \begin{pmatrix} u(t,x),\ t \ge 0,\ x \in \mathbb{R}^d, \\ u_t = u(t+\theta,x),\ t \ge 0,\ x \in \mathbb{R}^d,\ -r \le \theta \le 0 \end{pmatrix}$ with the norm $\|U(t,\cdot)\|_H = \left(\|u(t,\cdot)\|^2_{L_2^{\rho}(\mathbb{R}^d)} + \|u_t(\cdot)\|^2_{L_2([-r;0];L_2^{\rho}(\mathbb{R}^d))}\right)^{\frac{1}{2}}$.

We impose the following conditions.

(1) $\{f, \sigma\}\colon L_2([-r;0]; L_2^{\rho}(\mathbb{R}^d)) \to L_2^{\rho}(\mathbb{R}^d)$.

(2) The initial-datum function $\phi(t,x,\omega)\colon [-r,0]\times\mathbb{R}^d\times\Omega \to L_2^{\rho}(\mathbb{R}^d)$ is $\mathcal{F}_0$-measurable random function, independent of $W(t,x)$, $t \ge 0$, $x \in \mathbb{R}^d$, with almost surely

continuous paths and such that

$$\mathbf{E}\int\limits_{-r}^{0}\|\phi(t,\,\cdot\,)\|^p_{L_2^\rho(\mathbb{R}^d)}dt<\infty,\ \mathbf{E}\|\phi(0,\,\cdot\,)\|^p_{L_2^\rho(\mathbb{R}^d)}<\infty,\ p>2.$$

(3) $\{f,\sigma\}$ are such that

$\|f(u)\|_{L_2^\rho(\mathbb{R}^d)}+\|\sigma(u)\|_{L_2^\rho(\mathbb{R}^d)}\le L\big(1+\|u\|_{L_2([-r;0];L_2^\rho(\mathbb{R}^d))}\big)$, $u\in L_2([-r;0];L_2^\rho(\mathbb{R}^d))$,

$\|f(u)-f(v)\|_{L_2^\rho(\mathbb{R}^d)}+\|\sigma(u)-\sigma(v)\|_{L_2^\rho(\mathbb{R}^d)}\le L\|u-v\|_{L_2([-r;0];L_2^\rho(\mathbb{R}^d))}$,

$\{u,v\}\subset L_2([-r;0];L_2^\rho(\mathbb{R}^d))$.

We introduce the following definition.

Definition 1. A continuous random process $u(t,x,\omega)\colon H\to\mathbb{R}$ is called a **mild solution (solution)** to (1) – (1*) provided that

(1) It is $\mathcal{F}_t$-measurable for almost all $-r\le t\le T$.
(2) It satisfies the integral equation

$$u(t,x)=\int\limits_{\mathbb{R}^d}\mathcal{K}(t,x-\xi)\phi(0,\xi)d\xi+\int\limits_0^t\int\limits_{\mathbb{R}^d}\mathcal{K}(t-s,x-\xi)f(u_s)d\xi ds$$

$$+\int\limits_0^t\sum_{n=1}^{\infty}\sqrt{\lambda_n}\bigg(\int\limits_{\mathbb{R}^d}\mathcal{K}(t-s,x-\xi)\sigma(u_s)e_n(\xi)d\xi\bigg)d\beta_n(s),$$

$$0<t\le T,\ x\in\mathbb{R}^d, \tag{2}$$

$$u(t,x)=\phi(t,x),\ -r\le t\le 0,\ x\in\mathbb{R}^d,\ r>0. \tag{2*}$$

(3) It satisfies the condition $\mathbf{E}\int\limits_0^T\|u(t,\,\cdot\,)\|^p_{L_2^\rho(\mathbb{R}^d)}dt<\infty$, $p>2$, where $\mathscr{K}(t,x)=$ $=\begin{cases}\frac{1}{(4\pi t)^{\frac{d}{2}}}\exp\{-\frac{|x|^2}{4t}\}, & t>0,\\ 0, & t<0,\end{cases}$ denotes the fundamental solution (source function, diffusion kernel) of the heat equation.

Remark 1. It is assumed in the definition above that all the integrals in (2) are well defined.

We have proved that, under assumptions above, there exists a unique solution to the problem under investigation. We have also shown its Markovian and Feller property, and obtained sufficient conditions of invariant measure existence in terms of coefficients (coefficient conditions).

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FOURIER AND FOURIER-HAAR SERIES OF STOCHASTIC MEASURES AND THEIR APPLICATIONS

NELIA STEFANS'KA

Consider a mild solution of the Cauchy problem to the wave equation

$$\begin{aligned} u(t,x) &= \frac{1}{2}(u_0(x+at)-u_0(x-at)) + \frac{1}{2a}\int_{x-at}^{x+at} v_0(y)\,dy \\ &+ \frac{1}{2a}\int_0^t ds \int_{x-a(t-s)}^{x+a(t-s)} f(s,y,u(s,y))\,dy + \frac{1}{2a}\int_{(0,t]} d\mu(s)\int_{x-a(t-s)}^{x+a(t-s)} \sigma(s,y), \end{aligned} \tag{1}$$

where $(t,x)\in[0,1]\times\mathbb{R}$, $a>0$, μ is an SM defined on Borel σ-algebra $\mathcal{B}((0,1])$.

The integrals of random functions with respect to dx are taken for each fixed $\omega\in\Omega$.

We impose the following assumptions.

A1. Functions $u_0(y)=u_0(y,\omega):\mathbb{R}\times\Omega\to\mathbb{R}$ and $v_0(y)=v_0(y,\omega):\mathbb{R}\times\Omega\to\mathbb{R}$ are measurable and bounded for every fixed $\omega\in\Omega$.

A2. The function $f(s,y,v):[0,1]\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is measurable and bounded.

A3. $|f(s,y_1,v_1)-f(s,y_2,v_2)|\le L_f\left(|y_1-y_2|+|v_1-v_2|\right)$.

A4. The function $\sigma(s,y):[0,1]\times\mathbb{R}\to\mathbb{R}$ is measurable and bounded.

A5. $|\sigma(s_1,y_1)-\sigma(s_2,y_2)|\le L_\sigma\big(|s_1-s_2|^{\beta(\sigma)}+|y_1-y_2|^{\beta(\sigma)}\big),\quad 1/2<\beta(\sigma)\le 1.$

A6. For some random constant $C_\mu(\omega)$ $|\mu((0,t])|\le C_\mu(\omega),\ t\in(0,1]$.

For SM μ we consider the Fourier series $\sum_{k\in\mathbb{Z}}\xi_k\exp\{2\pi ikt\}$, $\xi_k=\int_{(0,1]}\exp\{-2\pi ikt\}\,d\mu(t)$, and its partial sums $S_j(t)=\sum_{|k|\le j}\xi_k\exp\{2\pi ikt\}$.

Denote

$$\begin{aligned} u_j(t,x) &= \frac{1}{2}(u_0(x+at)-u_0(x-at)) + \frac{1}{2a}\int_{x-at}^{x+at} v_0(y)\,dy \\ &+ \frac{1}{2a}\int_0^t ds \int_{x-a(t-s)}^{x+a(t-s)} f(s,y,u_j(s,y))\,dy + \frac{1}{2a}\int_{(0,t]} S_j(s)\,ds\int_{x-a(t-s)}^{x+a(t-s)} \sigma(s,y)\,dy\,. \end{aligned} \tag{2}$$

Theorem 1. *Let A1 – A6 be fulfilled, and assume that the following conditions hold: if $h\in\mathsf{L}_2((0,1])$ then h is integrable w. r. t. μ, and*

$$\text{if}\quad \int_{(0,1]}|h_j(x)|^2\,dx\to 0,\ j\to\infty\quad\text{then}\quad\int_{(0,1]}h_j(x)\,d\mu(x)\xrightarrow{\mathsf{P}}0,\ j\to\infty.$$

Then u from (1) and u_j from (2) have versions such that for every $0<\delta<1$

$$\sup_{x\in\mathbb{R},t\in[0,1-\delta]}|u_j(t,x)-u(t,x)|\xrightarrow{\mathsf{P}}0,\quad j\to\infty.$$

Changing the integrator by Fejèr sums $\tilde{S}_j(t)=\frac{1}{j+1}\sum_{0\le k\le j}S_k(t)$ we also obtain the approximations of solution u of equation (1).

Let $\chi_n(x)$ be the classical orthonormal Haar system in $(0,1]$. We consider the process $\tilde{\mu}(t)=\mu((0,t])$, $0\le t\le 1$.

For $2^k+1\le n\le 2^{k+1}$, the Fourier-Haar coefficients of μ have the form

$$\eta_n = 2^{k/2}\Big(-d_k^i\tilde{\mu}\big(d_k^i\big)+2d_{k+1}^{2i-1}\tilde{\mu}\big(d_{k+1}^{2i-1}\big)-d_k^{i-1}\tilde{\mu}\big(d_k^{i-1}\big)-\int_{\left(d_k^{i-1},d_{k+1}^{2i-1}\right]} t\,d\mu+\int_{\left(d_{k+1}^{2i-1},d_k^{i}\right]} t\,d\mu\Big),$$

where $d_k^i=i2^{-k}$, $k\ge 0$, $0\le i\le 2^k$. Using the partial sums of Fourier – Haar series $S_N(x)=\sum_{n=1}^N\eta_n\chi_n(x)$, we obtained approximation of trajectories μ.

Theorem 2. *For each $x\in(0,1]$, if $\mu(\{x\})=0$ a.s. then $S_N(x)\xrightarrow{\mathsf{P}}\tilde{\mu}(x)$, $\quad N\to\infty$.*

For continuous $\tilde{\mu}$ the uniform convergence of S_N holds.

We consider a mild solution of the Cauchy problem to the wave equations

$$\begin{aligned}u_j(t,x)&=\frac{1}{2}(u_0(x+at)-u_0(x-at))+\frac{1}{2a}\int_{x-at}^{x+at}v_0(y)\,dy\\&+\frac{1}{2a}\int_0^t ds\int_{x-a(t-s)}^{x+a(t-s)}f(s,y,u_j(s,y))\,dy+\frac{1}{2a}\int_{(0,t]}d\mu_j(s)\int_{x-a(t-s)}^{x+a(t-s)}\sigma(s,y)\,dy\,.\end{aligned}$$

We obtained the convergence of u_j provided that $\tilde{\mu}_j(t)=\mu_j((0,t])$ converge uniformly in probability.

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SOME ESTIMATION OF HURST PARAMETER IN MEASUREMENT ERROR MODEL

OLGA SYNIAVSKA

Let $\{\xi(t), t \in [0,1]\}$ be a fractional Brownian motion with Hurst parameter $H \in (0,1)$. We suppose that H is unknown and verifies $H^* \leq H < 1$, with H^* known.

Consider the following model with errors–in–variables. For a fixed $n \geq 1$ assume that we observe the values $X(0), X\left(\frac{1}{n}\right), \ldots, X(1)$, which differ from the true values of fractional Brownian motion $\{\xi(t), t \in [0,1]\}$ at the points

$$\left\{\frac{k}{n}\big|0 \leq k \leq n\right\}.$$

These differences are the measurement errors $\{\delta_{k,n}\big|0 \leq k \leq n\}$, which do not depend on fractional Brownian motion values $\{\xi\left(\frac{k}{n}\right)\big|0 \leq k \leq n\}$. More precisely,

$$X\left(\frac{k}{n}\right) = \xi\left(\frac{k}{n}\right) + \delta_{k,n}.$$

Suppose that $\delta_{k,n}$ are i.i.d. Gaussian random variables such that $\delta_{k,n} \simeq N(0,\sigma^2)$ and σ^2 is fixed.

Consider the Baxter sums sequences [1]:

$$S_n = \sum_{k=0}^{n-1}\left(X\left(\frac{k+1}{n}\right) - X\left(\frac{k}{n}\right)\right)^2 - 2n\sigma^2, n \geq 1.$$

Theorem 1. *Let $H^* \leq H < 1$, with $H^* < 1$ known. Then the interval $(I_l(n), I_r(n)) \cap (0,1)$ is the confidence interval for Hurst parameter H with the confidence level $1-p \in (0,1)$, where*

$$I_l(n) = \frac{1}{2}\left(1 + \frac{\ln(1-\varepsilon_n) - \ln S_n}{\ln n}\right),\ I_r(n) = \frac{1}{2}\left(1 + \frac{\ln(1+\varepsilon_n) - \ln S_n}{\ln n}\right),$$

$$\varepsilon_n \geq \sqrt{\frac{D_n(H^*)}{p}}, \quad D_n(H^*) = \frac{10}{n} + 8n^{2H^*-1}\sigma^2 + 8n^{4H^*-1}\left(1-\frac{1}{n}\right)\sigma^4 +$$

$$+\begin{cases} \frac{2}{n}\,\zeta(4-4H^*), & H^* \in (0,\frac{3}{4}); \\ \frac{2}{n}(1+\ln n), & H^* = \frac{3}{4}; \\ \frac{2}{n}\left(1+\frac{n^{4H^*-3}}{4H^*-3}\right), & H^* \in \left(\frac{3}{4},1\right), \end{cases} \qquad \zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}, s > 1.$$

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SIMULATION OF STOCHASTIC FIELDS AND TESTING HYPOTHESES ABOUT THEIR COVARIANCE FUNCTIONS.

V. TROSHKI, N. TROSHKI

Let $X = \{X(t,x), t \in \mathbb{R}, x \in [0,2\pi]\}$ be a mean square continuous real Gaussian homogeneous and isotropic stochastic field on $\mathbb{R}^2$.

$$X(t,x) = \sum_{k=1}^{\infty} \cos(kx) \int_0^{\infty} J_k(t\lambda) d\eta_{1,k} + \sum_{k=1}^{\infty} \sin(kx) \int_0^{\infty} J_k(t\lambda) d\eta_{2,k},$$

where $\eta_{i,k}, i = 1,2, k = \overline{1,\infty}$ are independent Gaussian processes with independent increments, $\mathbf{E}\eta_{i,k}(\lambda) = 0$, $\mathbf{E}(\eta_{i,k}(b) - \eta_{i,k}(c))^2 = F(b) - F(c), b > c$, $F(\lambda)$ is the spectral function. Let $J_k(u) = \frac{1}{\pi} \int_0^{\pi} \cos(k\varphi - u \sin\varphi) d\varphi$ be the Bessel functions of the first kind. Let $B(r) = 2^{\frac{n-2}{2}} \Gamma\left(\frac{n}{2}\right) \int_0^{+\infty} \frac{J_{\frac{n-2}{2}}(\lambda r)}{(\lambda r)^{\frac{n-2}{2}}} d\Phi(\lambda)$ be the covariance function of the field. The process

$$\hat{X}(t,x) = \sum_{k=1}^{M} \cos(kx) \sum_{l=0}^{N-1} \eta_{1,k,l} J_k(t\zeta_l) + \sum_{k=1}^{M} \sin(kx) \sum_{l=0}^{N-1} \eta_{2,k,l} J_k(t\zeta_l),$$

is considered as a model of the field $X(t,x)$ where $\eta_{i,k,l}, i = 1,2$ are independent Gaussian random variables, $\eta_{i,k,l} = \int_{\lambda_l}^{\lambda_{l+1}} d\eta_{i,k}(\lambda)$ are such that $\mathbf{E}\eta_{i,k,l} = 0$, $\mathbf{E}\eta_{i,k,l}^2 = F(\lambda_{l+1}) - F(\lambda_l) = b_l^2$, $\zeta_l, l = 0, \ldots, N-2$ are independent random variables being independent of $\eta_{i,k,l}$ and assuming values in the intervals $[\lambda_l, \lambda_{l+1}]$, $\zeta_{N-1} = \Lambda$, $b_l^2 > 0$ are such that

$$F_l(\lambda) = P\{\zeta_l < \lambda\} = \frac{F(\lambda) - F(\lambda_l)}{F(\lambda_{l+1}) - F(\lambda_l)}.$$

Accuracy and reliability of the model in the space $L_p(\mathbb{T}), p \geq 1$.

Theorem 1. *Let $\frac{1}{2} < \alpha \leq 1$ and let $\int_0^{\infty} \lambda^{2\alpha} dF(\lambda) < \infty$. Assume that a partition L used to construct a model $\hat{X}(t,x)$, $t \in [0,T], x \in [0,2\pi]$, is such that $I \leq \frac{\beta^p}{\max\left(\left(2\ln\frac{2}{\delta}\right)^{\frac{p}{2}}, p^{\frac{p}{2}}\right)}$, where*

$$I = \frac{T^{p\alpha+1}}{p\alpha+1} \left(\frac{2^p D_p^3 M^{\frac{p}{2}}}{(2\alpha-1)^{\frac{p}{2}}} \left(2\alpha - \frac{1}{M^{2\alpha-1}}\right)^{\frac{p}{2}} 2^{\frac{p}{2}+1} \cdot 4^{p(1-\alpha)} \pi^{p\alpha+1} \left(\sum_{l=0}^{N-2} |\lambda_{l+1} - \lambda_l|^{2\alpha} b_l^2 \right)^{\frac{p}{2}} + \right.$$
$$\left. + D_p 2^{p(1-\alpha)+1} \pi^{p\alpha+1} \left(\frac{4}{(2\alpha-1)M^{2\alpha-1}}\right)^{\frac{p}{2}} \left(\int_0^{\infty} \lambda^{2\alpha} dF(\lambda) \right)^{\frac{p}{2}} \right) + \frac{T^{2p\alpha+1}}{2p\alpha+1} \cdot \frac{2^p D_p^3 M^{\frac{p}{2}}}{(2\alpha-1)^{\frac{p}{2}}} \left(2\alpha - \frac{1}{M^{2\alpha-1}}\right)^{\frac{p}{2}} \times$$
$$\times 2^{\frac{p}{2}+1} 4^{p(1-\alpha)} \pi^{p\alpha+1} \left(\frac{1+C}{2}\right)^{p\alpha} \left(\sum_{l=0}^{N-2} |\lambda_{l+1} - \lambda_l|^{2\alpha} \int_{\lambda_l}^{\lambda_{l+1}} \lambda^{2\alpha} dF(\lambda) \right)^{\frac{p}{2}} +$$
$$+ T \cdot 2^{2p+1} \pi D_p^2 M^p (F(+\infty) - F(\Lambda))^{\frac{p}{2}},$$

and where $C = \max\limits_{0<l\le N-2} \frac{\lambda_{l+1}}{\lambda_l}$, $D_p = \begin{cases} 1, & \textit{if } 0 < \frac{p}{2} \le 1, \\ 2^{\frac{p}{2}-1}, & \textit{if } \frac{p}{2} > 1 \end{cases}$. *Then the model* $\hat{X}(t,x)$ *approximates the Gaussian field* $X(t,x)$ *with reliability* $1-\ \delta, 0<\delta<1$, *and accuracy* $\beta > 0$ *in the space* $L_p(\mathbb{T}), p \ge 1$.

Construction of a criterion for testing hypothesis. Let $\mathbb{H}$ be the hypothesis that the covariance function of homogeneous and isotropic continuous in mean square Gaussian stochastic field equals $B(r)$ for $0 \le r \le A$.

Denote

$$g(\varepsilon) = 2\sqrt{1 + \frac{\varepsilon^{1/p}\sqrt{2}}{C_p^{\frac{1}{p}}} \exp\left\{-\frac{\varepsilon^{\frac{1}{p}}}{\sqrt{2}C_p^{\frac{1}{p}}}\right\}}.$$

Let $\varepsilon \ge z_p = C_p\left(\frac{p}{\sqrt{2}} + \sqrt{(\frac{p}{2}+1)p}\right)^p$ and ε_δ be a solution of the equation $g(\varepsilon) = \delta$, $0<\delta<1$. Put $S_\delta = \max\{\varepsilon_\delta, z_p\}$. It is obviously that $g(S_\delta) \le \delta$ and

$$P\left\{\int\limits_0^A (\hat{B}(r) - B(r))^p dr > S_\delta\right\} \le \delta. \tag{1}$$

From the paper [3] and (1) it follows that to test the hypothesis $\mathbb{H}$ one can use the following criterion.

Criterion 1. *For a given level of confidence* δ *the hypothesis* $\mathbb{H}$ *is accepted if*

$$\int\limits_0^A (\hat{B}(r) - B(r))^p d\mu(r) < S_\delta,$$

otherwise hypothesis is rejected.

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CONVERGENCE OF SOLUTIONS OF SDES TO COALESCING HARRIS FLOWS

M. B. VOVCHANSKII

Let $\mathbb{D}(R)$ be a separable topological space of nondecreasing right-continuous functions on R equipped with the Skorokhod topology.

Definition 1. A Harris flow [1, 3, 4] X with the infinitesimal covariance φ is a family of $\mathbb{D}(R)-$valued random variables $\{X(s,t) \equiv X(\cdot,s,t) \mid s \le t\}$ such that

(1) for any $s \le t \le r$ $P\{X(\cdot,s,r) = X(\cdot,t,r) \circ X(\cdot,s,t)\} = 1$; $X(s,s) = Id$ a.s.;
(2) for any $t_1 \le t_2 \le \ldots \le t_n$ random elements $X(t_1,t_2),\ldots,X(t_{n-1},t_n)$ are independent;
(3) for any $s,t \in \mathbb{R}, h > 0$ $Law(X(s,t)) = Law(X(s+h,t+h))$;
(4) $X(0,h) \to Id$ in probability as $h \to 0+$;
(5) for any x the process $t \mapsto X(x,0,t) - x$ is a Brownian motion started at 0 w.r.t. the filtration $\{X(u_1,u_2), 0 \le u_1 \le u_2 \le t\}_{t \ge 0}$;
(6) for any x, y

$$\langle X(x,0,\cdot), X(y,0,\cdot)\rangle(t) = \int_0^t \varphi(X(x,0,s) - X(y,0,s))ds.$$

A sequence of Harris flows with infinitesimal covariances φ_n converges as diffusions [2] to a Harris flow with infinitesimal covariance φ, as the functions φ_n converge to φ uniformly on compact sets [1]. A stronger result is presented in the talk. We consider a particular case of a Harris flow X whose infinitesimal covariance is a characteristic function φ of a symmetric $\alpha-$stable distribution with $\alpha \in (0;2)$.

If $\int_0^\delta \frac{x}{1-\varphi(x)}dx$ is finite for any small positive δ a Harris flow can be referred to as a coalescing flow in the sense that any two Brownian particles carried by the flow meet with probability 1 and stick together after the collision. In this case a mapping $X(\cdot,s,t)$ is discontinuous. However, one can still consider an inverse flow [1, 5] defined via

$$X^{-1}(x,s,t) = \sup\{y\colon X(y,r,t) \le x, r \le s\}.$$

Let $\mathbb{C}([a;b])$ be a space of continuous functions on $[a;b]$ equipped with the topology of uniform convergence, and $\mathbb{M}(\mathbb{R})$ be a space of locally finite Radon measures on the real line equipped with the vague topology. In product spaces the product topology is always considered.

Theorem 1. *Suppose that $\{\varphi_n\}_{n\in\mathbb{N}}$ is a sequence of symmetric infinitely differentiable nonnegative functions such that φ_n converge to φ uniformly on compact sets as $n \to \infty$. For each n, define a stochastic flow $\{X_n(\cdot,s,t) \mid 0 \le s \le t\}$ as a solution to the following SDE:*

$$X_n(x,s,t) = x + \int_s^t F_n(X_n(x,s,r), dr),$$

where $F_n = \{F_n(x,t) \mid x \in \mathbb{R}, t \in \mathbb{R}^+\}$ *is a centered Gaussian field with covariance* $\min\{t,s\}\varphi_n(x-y)$. *Then*

(1) *for any* $x_1,\ldots,x_N, x_i \in \mathbb{R}, i=\overline{1,N}, N\in\mathbb{N}, 0\le s\le t$

$$\big(X_n(x_1,s,\cdot),\ldots,X_n(x_N,s,\cdot),X_n^{-1}(x_1,\cdot,t),\ldots,X_n^{-1}(x_N,\cdot,t)\big) \Rightarrow$$
$$\big(X(x_1,s,\cdot),\ldots,X(x_N,s,\cdot),X^{-1}(x_1,\cdot,t),\ldots,X^{-1}(x_N,\cdot,t)\big), n\to\infty,$$

in $(\mathbb{C}([s;t]))^{2N}$;

Let λ *be the Lebesque measure on the real line. For* $n\in\mathbb{N}, 0\le s\le t$, *define the following* $\mathbb{M}(\mathbb{R})-$*valued random elements:*

$$\mu_n(s,t)=\lambda\circ X_n(\cdot,s,t)^{-1}, \mu(s,t)=\lambda\circ X(\cdot,s,t)^{-1},$$
$$\hat{\mu}_n(s,t)=\lambda\circ X_n^{-1}(\cdot,s,t)^{-1}, \hat{\mu}(s,t)=\lambda\circ X^{-1}(\cdot,s,t)^{-1}.$$

Then

(2) *for any* $s_1\le,\ldots\le s_N, s_t\le,\ldots\le t_N, s_i\le t_i, i=\overline{1,N}, N\in\mathbb{N}$

$$(\mu_n(s_1,t_1),\ldots,\mu_n(s_N,t_N),\hat{\mu}_n(s_1,t_1),\ldots,\hat{\mu}_n(s_N,t_N))$$
$$\Rightarrow(\mu(s_1,t_1),\ldots,\mu(s_N,t_N),\hat{\mu}(s_1,t_1),\ldots,\hat{\mu}(s_N,t_N)), n\to\infty,$$

in $(\mathbb{M}(\mathbb{R}))^{2N}$.

The nature of noises associated with Harris flows [4, 6] implies that for $\alpha\in(0;1)$ the weak convergence in Theorem 1 cannot be replaced with a stronger one.

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NEW RESULTS ON ESTIMATION OF IMPULSE RESPONSE FUNCTIONS

V. ZAIATS, I. BLAZHIEVSKA

Identification and estimation of linear systems, in a parametric or non-parametric framework, is often based on “black-box” models. The input to a black box may be single or multiple, perfect or noisy, and the same happens to the output. These models arise in different disciplines within engineering and science, such as control, communications and networks, signal processing, biology. An important feature of any linear system is that it is uniquely identified by means of its impulse response function. In Chapter 5 of their classical book [1], Gikhman and Skorokhod mentioned the possibility of using linear transformations of stochastic processes for statistical analysis of “black-box” models. In this presentation, we consider a single input–double output channel model described by a linear time-invariant system whose IRF has two L_2-integrable components. Then the outputs belong to the class of Wiener shot noise processes and include solutions of many stochastic differential equations under a fixed choice of the kernels. We assume that IRF in one channel is unknown while it is known in another channel and focus on estimation of the unknown IRF after observations of the outputs in both channels. Different deterministic and statistical approaches to this problem have been used. In a cornerstone monograph [2], a “polynomial” representation for systems with Gaussian white noise inputs was set down and analysis–synthesis of these systems was discussed. Identification or estimation problems for polynomial systems using Gaussian inputs can be formulated in terms of Fourier/Laplace transforms. In both approaches, the system was supposed to be stable requiring the kernels to be L_1-integrable.

The papers [3], [4], [5], [6] [7] have successfully removed the restriction of Gaussian white noise inputs and that of system’s stability. The approach in these papers was based on: (i) a specific approximation of Gaussian white noise inputs, and (ii) cross-correlating system’s input and output. It was applied to estimation of L_2-integrable IRFs in linear time-invariant (LTI) single input–single output (SISO) systems. Further, theory of square-Gaussian processes has been used in assessing estimator’s supremum and to test hypothesis on IRFs; see [8], [9], and [10]. In our presentation, we would like to extend these results to SIDO-systems.

The choice of our approach is motivated by an attempt to set a link between three theories:

- L_2-theory for kernels of stochastic integrals;
- cumulant analysis applied to sample cross-correlograms;
- theory of integrals involving cyclic products of kernels.

The next step is to apply cumulant analysis of sample cross-correlorams of Gaussian stationary processes. Here, an important role is played by integral representations for higher-order cumulants driven by the estimator of the unknown IRF. The well-known combinatorial diagram formulae enable us to reduce the cumulants to finite sums of multi-dimensional integrals involving cyclic products of kernels and dependent on parameters.

Convergence to zero of these integrals leads to asymptotic normality of the estimator. Our proof of the CLT is based on the theory of integrals involving cyclic products of kernels, see [5], rather then on the theory of Delta matroid integrals developed in [11], [12], [13]. The reason is in the desire to keep unaltered the weakest possible L_2-restriction on the unknown IRF.

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THE RATE OF GROWTH OF THE COMPOUND RENEWAL PROCESSES AND THEIR INCREMENTS

N. M. ZINCHENKO

Let $D(t)$ be a *compound renewal processes* (random sum, randomly stopped sum) of the form

$$D(t) = S(N(t)) = \sum_{i=1}^{N(t)} X_i, \tag{1}$$

where $\{X_i, i \geq 1\}$ are r.v., $S(t) = \sum_{i=1}^{[t]} X_i$, $t > 0$, $S(0) = 0$; $\{Z_i, i \geq 1\}$ is another sequence of non-negative r.v. independent of $\{X_i\}$, $Z(x) = \sum_{i=1}^{[x]} Z_i$, $x > 0$, $Z(0) = 0$ and *renewal (counting) process* $N(t)$ is defined as $N(t) = \inf\{x \geq 0 : Z(x) > t\}$.

Our main task is to find the conditions on $\{X_i\}$ and $\{Z_i\}$ and the form of normalizing function $f(t)$ and the centering function $m(t)$, for which a.s.

$$\limsup_{t\to\infty} \left(D(t) - m(t)\right)/f(t) = const \text{ and/or } \liminf_{t\to\infty} \left(D(t) - m(t)\right)/f(t) = const.$$

We proposed a number of integral tests for investigation the rate of growth of the compound renewal processes $D(t)$ as $t \to \infty$. The cases of independent, weakly dependent and associated summands are studied as well as random variables satisfying φ-mixing conditions. For example, when both $\{X_i\}$ and $\{Z_i\}$ are i.i.d.r.v. and have moments of order $p \geq 2$, then non-decreasing function $f(t) = ct^{1/2}h(t)$, $h(t) \uparrow \infty$, $c > 0$, will be an upper (lower) function for centered process $(D(t) - m\lambda t)$, $EX_1 = m$, $EZ_1 = 1/\lambda$, according to convergence (divergence) of the integral

$$\int_1^\infty t^{-1}h(t)\exp\{-h^2(t)/2\}dt.$$

In the case when i.i.d.r.v. $\{X_i\}$ are attracted to asymmetric stable law $G_{\alpha_1,-1}$, the problem connected with upper (lower) functions for centered process $(D(t) - m\lambda t)$ is solved with the help of integral

$$\int_1^\infty t^{-1}h^{-\theta_1/2}(t)\exp\{-Bh^{\theta_1}(t)\}dt,$$

where

$$B = B(\alpha_1) = (\alpha_1 - 1)\alpha_1^{-\theta_1}|\cos(\pi\alpha_1/2)|^{1/(\alpha_1-1)}, \quad \theta_1 = \alpha_1/(\alpha_1 - 1).$$

Mentioned integral tests give the possibility to prove various modifications of the LIL for $D(t)$, for instance, we have

Theorem 1. *(Classical LIL for compound renewal processes). Let $\{X_i\}$ and $\{Z_i\}$ be independent sequences of i.i.d.r.v. with $EX_1 = m < \infty$, $0 < EZ_1 = 1/\lambda < \infty$, $\sigma^2 = VarX_1 < \infty$, $\tau^2 = VarZ_1 < \infty$. Then a.s.*

$$\limsup_{t\to\infty} \frac{|D(t) - m\lambda t|}{\sqrt{2t\ln\ln t}} = \nu, \quad \nu^2 = \lambda\sigma^2 + \lambda^3 m^2\tau^2. \tag{2}$$

Corresponding proofs are based on rather general theorems about the strong approximation of the random sums by a Wiener or α-stable Lévy process under various moment and dependence assumptions on $\{X_i\}$ and $\{Z_i\}$ [1].

Integral tests also occur to be useful when investigating the rate of growth of increments

$$D(t+a_t)-D(t)=\sum_{i=N(t)+1}^{N(t+a_t)} X_i \tag{3}$$

on the intervals, whose length a_t grows as $t\to\infty$, but not faster than t. As a consequence various modifications of the Erdös-Rényi-Csörgő-Révész-type SLLN for compound renewal processes were obtained.

Mentioned results, besides pure theoretical, have also certain practical interest, since compound renewal processes are successfully used in actuarial mathematics in various risk models: in rather popular *Sparre-Anderssen collective risk model* random sum $D(t) = S(N(t))$ is interpreted as a total claim amount arising during time interval $[0,t]$ and its increments $D(t+a_t)-D(t)$ - as claim amount during time interval $[t,t+a_t]$; in *classical Cramér-Lundberg risk model* $D(t)$ is a compound Poisson process with intensity $\lambda > 0$. Thus, our results describe the rate of growth and fluctuations of the reserve capital and total claim payments. Finding the non-random boundaries $f(t)$ for the rate of growth of $D(t)$ and their increments helps to analyze and plan insurance activities and to specificate the value of insurance premiums and reserves [2].

The case of risk models with stochastic premiums [3] is also interesting but more complicated for investigation, some results for such models are also obtained.

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