LIMIT THEOREMS IN PROBABILITY THEORY, NUMBER THEORY AND MATHEMATICAL STATISTICS

INTERNATIONAL WORKSHOP IN HONOUR OF PROF. V.V. BULDYGIN

ABSTRACTS

October 10-12, 2016
NTUU «KPI»
Kyiv, Ukraine

This Workshop[1] continues the series of events previously held in Cologne (2010), Paderborn (2011), Kyiv (2011), Ulm (2012) and Uppsala (2013).

Programme Committee

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Programme

	Talks a.m.	Lunch break	Talks p.m.	Conf. dinner
Mon, Oct 10	9:30-12:40	12:40-14:00	14:00-18:00	19:00-21:00
Tue, Oct 11	9:30-13:00	13:00-14:30	14:30-18:00	
Wed, Oct 12			14:10-18:00	

[1]DFG Project GZ: lN 23117-1 "A unified approach to limit theorems for dual objects in probability and number theory"

Contents

MATHEMATICAL RESEARCH ACHIEVEMENTS OF V. V. BULDYGIN

K.-H. INDLEKOFER, O. I. KLESOV, J. G. STEINEBACH

We provide a brief account of life and mathematical achievements of Prof. V. V. Buldygin in this talk.

V. V. Buldygin
(05.11.1946–17.04.2012)

V. V. Buldygin was born on November 5, 1946 in Tbilisi.

In 1965, he entered Taras Shevchenko Kiev National University and graduated from the Faculty for Mechanics and Mathematics in 1970.

He was granted the gold medal of the Ministry of Education of the Soviet Union for his graduation theses.

After obtaining his Ph.D. in 1973 from the Department of Probability Theory and Mathematical Statistics of Taras Shevchenko Kiev National University, he joined the Institute of Mathematics of the National Academy of Science of Ukraine and worked under the supervision of Anatolii Volodymyrovych Skorokhod till 1986. In 1982, he obtained the degree of Doctor in Physics and Mathematics (equivalent of Dr. hab.).

V. V. Buldygin was appointed as the Head of the Department of Mathematical Analysis and Probability Theory of the National Technical University of Ukraine "Kyiv Polytechnic Institute" in 1986 and guided this department till his death. He was promoted to Professor in 1987.

The field of his mathematical investigations was very broad. In the 1980th, his scientific interests, as a researcher of Institute of Mathematics, were focused on the convergence of random elements in linear spaces. Well known are his results on the sums of independent random elements in Banach spaces (a generalization of the Lévy inequality, comparison principle for series of independent terms, contraction principle for Gaussian random variables, etc.). His doctoral dissertation *"Convergence of Random Elements in Topological Spaces"* was published in 1980 [1]. In 1985, he coauthored another monograph *"The Brunn–Minkowski inequality and its applications"* (written jointly with A. Kharazishvili) [2]. The latter monograph was translated into English by Kluwer Academic Publishers in 2000.

In the 1970s, he started to study the concept of sub-Gaussian distributions. His results on this topic are summarized in the monograph *"Metric characterization of random variables and random processes"* written jointly with Yu. V. Kozachenko (the Russian edition is published by TBiMC in 1998, and the English edition appeared in 2000 via AMS).

Simultaneously, he developed the methods for studying the oscillatory properties of Gaussian sequences and the limit theorems for sums of independent random terms with operator normalizations. These results were included in his monograph *"Functional methods in problems of the summation of random variables"* [3] written jointly with S. A. Solntsev (the Russian edition appeared in 1989; it was translated into English in 1997 by Kluwer Academic Publishers).

In the 1990s, he studied the statistical properties of the estimators for correlation functions of stationary Gaussian stochastic processes and random fields, asymptotic behavior of solutions of stochastic difference equations, exponential estimates for the distributions of maxima of stochastic processes. Other fields of his interests covered, in particular, the statistical estimates of impulse transfer functions for linear systems, properties of empirical correlograms, and the Lévy-Baxter theorems for shot-noise processes.

In the last decade, his interests switched to studying the so-called generalized renewal processes and the corresponding classes of functions. He introduced the so-called functions with a group of regular points and proved an analog of Karamata's representation theorems for them. His last monograph *"Pseudo-Regularly Varying Functions and Generalized Renewal Processes"* [5] appeared soon after his death.

Renewal theory is a branch of probability theory rich of fascinating mathematical problems and also of various important applications. On the other hand, regular variation of functions is a property that plays a key role in many fields of mathematics. One of the main aims of [5] is to exhibit some fruitful links between these two areas via a generalized approach to both of them. The interest in generalizing the notion of Karamata's regular variation was stimulated by some applications to certain asymptotic problems in renewal theory and can be traced back to the very first days of this century.

Generally, we call two objects *dual* if they are inverse to each other in some sense and their asymptotic properties are related to each other, that is, if a limit result for the first object implies a corresponding one for the second object, and vice versa. The duality of the renewal process $\{N_t\}$ and the corresponding sequence of sums of random variables $\{S_n\}$ is the starting point for researches presented in [5].

Some problems on asymptotic behavior of sequences of sums whose terms are elements of one-dimensional and multi-dimensional linear regressions with independent and symmetric noise are studied in [6].

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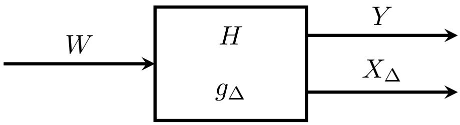
PROPERTIES OF ESTIMATORS OF RESPONSE FUNCTIONS IN TWO-DIMENSIONAL SYSTEMS

I. P. BLAZHIEVSKA

Let us consider a "black box":



which is described by the linear two-dimensional system. Its response function has real-valued components – an unknown function $H = (H(\tau), \tau \geq 0)$ and a known function $g_\Delta = (g_\Delta(\tau), \tau \geq 0), \Delta > 0$. We suppose that $H \in L_2(\mathbf{R})$, and g_Δ satisfies some general properties. The system has two observable outputs:

$$Y(t) = \int_{-\infty}^{t} H(t-s)dW(s), t \in \mathbf{R};$$

$$X_\Delta(t) = \int_{-\infty}^{t} g_\Delta(t-s)dW(s), t \in \mathbf{R},$$

and the input $W = (W(t), t \in \mathbf{R})$ is a standard Wiener measure on $\mathbf{R}$.

We investigate the properties of the sample cross-correlogram

$$\widehat{H}_{T,\Delta}(\tau) = \frac{1}{T}\int_0^T Y(t+\tau)X_\Delta(t)dt, \ \tau \geq 0,$$

that is taken as the estimator of H, and apply some results from [1].

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MILD SOLUTION OF PARABOLIC EQUATION WITH STOCHASTIC MEASURE

I. M. BODNARCHUK

Let μ be a stochastic measure defined on Borel σ-algebra $\mathcal{B}(\mathbb{R})$, i.e. $\mu : \mathcal{B}(\mathbb{R}) \to L_0(\Omega, \mathcal{F}, \mathsf{P})$ is a σ-additive mapping.

Consider the stochastic parabolic equation in the following mild sense

$$u(t,x) = \int_{\mathbb{R}} p(t,x;0,y)u_0(y)\,dy + \int_0^t ds \int_{\mathbb{R}} p(t,x;s,y)f(s,y,u(s,y))\,dy + \int_{\mathbb{R}} d\mu(y) \int_0^t p(t,x;s,y)\sigma(s,y)\,ds\,. \tag{1}$$

Here $u(t,x) = u(t,x,\omega) : [0,T] \times \mathbb{R} \times \Omega \to \mathbb{R}$ is an unknown measurable random function, $p(t,x;s,y)$ is the fundamental solution of the operator

$$Lu(t,x) = a(t,x)\frac{\partial^2 u(t,x)}{\partial^2 x} + b(t,x)\frac{\partial u(t,x)}{\partial x} + c(t,x)u(t,x) - \frac{\partial u(t,x)}{\partial t},$$

where functions $a,\ b,\ c$ are defined on the set

$$S = [0,T] \times \mathbb{R} = \{(t,x) : t \in [0,T],\ x \in \mathbb{R}\}.$$

Under certain assumptions regarding the coefficients of the operator L and functions f, σ, u_0 we investigate the solution of (1).

The particular case of problem (1) (the heat equation) is investigated in [1]. Existence and uniqueness of the mild solution are proved in this publication. Hölder continuity of its paths in time and space variables is established. We extend this results to the case of parabolic equation and improve the Hölder continuity exponents.

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ONE APPROACH TO CONTROL OF LINEAR DYNAMICAL SYSTEMS

S. V. BODNARCHUK

Consider a linear differential equation of the form

$$x(t) = x + \int_0^t Ax(s)ds + B\gamma(t), \quad t \geq 0, \tag{1}$$

where $x(0) = x \in \mathbb{R}^m$, A and B are $m \times m$ and $m \times d$ matrices, respectively, $\gamma : \mathbb{R}_+ \to \mathbb{R}^d$ is a measurable bounded function, $x(\cdot) : \mathbb{R}_+ \to \mathbb{R}^m$ is an unknown function that displays the state of the dynamical system at time $t > 0$.

In the classical theory control problem is formulated as follows (see [1]): given the initial point x, does there exist such function γ that $x(t) = 0$ for some $t > 0$? Such a type of control allows to prove the ergodicity of solutions to SDE's of diffusion type (see [2, 3]).

We propose a different approach to the concept of control (see [4]): given the initial point x, does there exist such time transformation $\lambda : [0, +\infty) \to [0, +\infty)$ that solution of equation (1) with inhomogeneity $\gamma(\lambda(t))$ coincides, for some $T > 0$, with solution of this equation with $x = 0$ and inhomogeneity $\gamma(t)$? It appears that such a type of control allows to prove the ergodicity of solutions to SDE's driven by Lévy processes (see [5]).

In the report we present necessary and sufficient conditions for controllability by time transformation of the equation (1).

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DIAGONAL RANDOM OPERATOR IN A SEPARABLE HILBERT SPACE

O. O. DASHKOV

Consider $l_2 = \{x = (x_1, x_2, \dots) \,|\, x_k \in \mathbb{R}, k \geq 1, \sum_{k=1}^{\infty} x_k^2 < +\infty\}$ as real separable Hilbert space with inner product $(x, y) = \sum_{k=1}^{\infty} x_k y_k$ and norm $||x|| = \sqrt{\sum_{k=1}^{\infty} x_k^2}$. Let $e_k = (0, 0, \dots, 1, 0, \dots)$ (unit on the k-th position), $k \geq 1$ – standard orthonormal basis in l_2, $\xi_1, \xi_2, \dots$ be independent standard normal variables on probability space $(\Omega, \mathcal{F}, P)$.

In this paper we work with diagonal random operator A defined as follows

$$Ax = (\xi_k x_k)_{k=1}^{\infty},\ x = (x_k)_{k=1}^{\infty} \in l_2.$$

We show that for the set $K = \{x = (x_1, x_2, \dots) \in l_2 \,|\, \sum_{k=1}^{\infty} k x_k^2 \leq 1\}$ its image under operator A is well-defined and compact almost surely.

To investigate the properties of $A(K)$ associated with tending coordinate to infinity we consider orthogonal projective operators Q_n on the c.l.s. of $\{e_{n+1}, e_{n+2}, \dots\}$ defined as

$$Q_n x = (0, 0, \dots, 0, x_{n+1}, x_{n+2}, \dots),\ x = (x_n)_{n \geq 1} \in l_2,\ n \geq 1.$$

We prove that Q_n converges uniformly to zero on any compact set $C \subset l_2$.

We investigate the behavior of diameters of $Q_n A(K)$ and show that $diam(Q_n A(K)) = 2 \sup_{k>n} \frac{|\xi_k|}{\sqrt{k}}$.

Proposition 1.

$$\sup_{k>n} \frac{|\xi_k|}{\sqrt{k}} \sim \sqrt{\frac{2 \ln n}{n}},\ n \to \infty\ a.s.$$

While proving, we show that for $0 < c < 1,\ c^2 < \alpha < 1$ cardinality of the set $\{m \in \mathbb{N} : N - N^{\alpha} < m \leq N,\ |\xi_m| > c\sqrt{2 \ln m}\}$ tends to infinity when $N \to \infty$ almost surely.

Let $N_{\varepsilon}(B)$ denote the smallest number of the closed balls with radii ε that cover a compact B. We show, that for $R > r$ $N_r(\overline{B_R^d(0)}) \geq d+1$, where $\overline{B_R^d(0)}$ is a closed ball in $\mathbb{R}^d$ with radii R and center in zero. We use it to prove the last statement of the work:

Proposition 2. *Suppose that for a sequence* $\varepsilon_n \to 0,\ n \to \infty$ *the sequence* $a_n = N_{\varepsilon_n}(Q_n A(K))$ *satisfies conditions:*

1) $P(\overline{\lim_{n\to\infty}}\{a_n = 1\}) \neq 1$;

2) $P(\{a_n - \text{unbounded}\}) \neq 1$.

Then $\varepsilon_n \sim \sqrt{\frac{2\ln n}{n}},\ n \to \infty$.

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HILBERT-VALUED FUNCTIONS AND SELF-INTERSECTIONS OF GAUSSIAN PROCESSES

A. A. DOROGOVTSEV AND O. L. IZYUMTSEVA

In the talk we establish connections between the covariance function of the Gaussian process and properties of its self-intersection local time. We consider the process with the following covariance function

$$K(t,s) = (Ag_0(t), Ag_0(s)),\ \ s,t \in [0;1].$$

Here $g_0(t) = 1_{[0;t]},\ A$ is a continuous linear operator in $L_2([0;1])$. For a one-dimensional or a two-dimensional process we study the objects

$$\int_0^1 \delta_0(x(t))dt, \quad \int_{\Delta_n} \prod_{j=1}^{n-1} \delta_0(x(t_{j+1}) - x(t_j))d\vec{t},$$

where $\Delta_n = \{0 \le t_1 \le \ldots \le t_n \le 1\}$. The last expression is called by the self-intersection local time and must be renormalized. The main results establish the existence of the mentioned objects and give the estimates of its moments. As a tool we use the properties of the L_2-valued function $Ag_0(t),\ t \in [0;1]$.

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DISCRETE-TIME TRAWL PROCESSES

P. DOUKHAN

The talk essentially aims at describing the ongoing joint work [3] *with Silvia Lopes, Adam Jakubowski, and Donatas Surgailis.*

The classical infinite moving averages

$$X_k = \sum_{j=0}^{\infty} b_j \xi_{k-j}, \qquad \sum_{j=0}^{\infty} |b_j| = \infty, \qquad \sum_{j=0}^{\infty} b_j^2 < \infty,$$

defined through an iid sequence $(\xi_j)_j$ of random variable with $E\xi_0^2 < \infty, E\xi_0 = 0$ were shown to exhibit a long range dependent (LRD) behavior in a famous paper by Davydov (1970). If $b_j \sim cj^{-\beta}$ for $\frac{1}{2} < \beta < 1$ then var $S_n \sim cn^{-(3-2\beta)} \gg n$ if $S_n = X_1 + \cdots + X_n$.
A nice paper by Barndorff-Nielsen et al. (2014) [1], provides a nice continuous time extension of this paper in which the second order behavior of the above process is described and where an analogue distributional behavior is also predicted for integer valued models.
The present talk describes a discrete time version of such trawl processes provided by an iid sequence of processes $\gamma_k : R \to R$ and an analogue relation

$$X_k = \sum_{j=0}^{\infty} \gamma_{k-j}(a_j), \qquad a_j \sim cj^{-\alpha}, \qquad 1 < \alpha < 2$$

then we exhibit simple conditions such that L^2-LRD behavior holds with $H = (3-\alpha)/2$ and $\alpha = 2\beta$.
We also fit the parameter α . For this we quote that the covariance of this model satisfies $r_k \sim ck^{1-\alpha}$, thus estimates of the covariance are proved to satisfy $\widehat{r}_k/\widehat{r}_{[\delta k]} \to \delta^{\alpha-1}$. Consistency of such estimates is deduced from a weak dependence argument.
In case $\alpha > 2$ a standard Brownian limit occurs with normalization $\sqrt{n}$.
Two classes of such processes satisfy either the same behavior as above or $n^{-1/\alpha}(S_{[nt]} - ES_{[nt]})$ converges to a Lévy $\alpha-$stable process in some sense. This case includes integer valued LRD models, if γ is either a unit Poisson process or the Bernoulli process $\gamma(u) = \mathbf{1}_{\{U \le u\}}$ for a

uniform random variable U on $[0,1]$. A non trivial example for which the first behavior holds is $\gamma = W$, the Brownian motion.
If now the sequence a_j is non non-increasing then we also exhibit seasonal behaviors for models

$$X_k = \sum_{j=0}^{\infty} c_j \gamma_{k-j}(a_j), \qquad a_j \sim c j^{-\alpha}, \qquad 1 < \alpha < 2$$

with c_j a periodic sequence, see Bisognin and Lopes, in [4].

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STATISTICAL ANALYSIS OF TESTS IN HIGHER MATHEMATICS

A. A. DYKHOVYCHNYI AND A. F. DUDKO

This report informs about methods of quality analysis of tests in higher mathematics offered in the KPI.

The methods of modern statistics certainly form the basis of the analysis of the quality of educational tests in higher mathematics.

Traditionally, these methods are divided into methods of classical test theory (CTT) and modern methods of Item Response Theory (IRT). CTT is based on traditional methods of statistical analysis. The main idea of IRT-methods is introduction of two sets of latent parameters, namely set of person parameter and set of item parameters, that are related by certain probability logistic functions.

Note that the use of classical and modern methods isn't contradictory. At the same time combined application of CTT and IRT, interpretation and coordination of the results are of particular interest.

Thus, the mathematical problems of practical application of IRT-methods, creating of the methodology of test analysis and its software implementation are the essence of research in the field of statistical analysis of the test quality in higher mathematics, offered by the Department of Mathematical Analysis and Probability Theory of NTUU "KPI". The information about these researches is presented in [1] in more details.

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THE RATE OF WEAK CONVERGENCE OF THE N-POINT MOTIONS OF HARRIS FLOWS

V. V. FOMICHOV

Let $\{x(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ be a Harris flow with covariance function Γ, which has compact support. If the function Γ is smooth enough, this Harris flow can be represented as a flow of solutions of some stochastic differential equation. For this case it was shown in [1] that when the diameter $d(\Gamma)$ of the support of the function Γ tends to zero the n-point motions of the Harris flow converge weakly to the n-point motions of the Arratia flow $\{x_0(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ (recall that the Arratia flow is a Harris flow with covariance function $1\!\!\mathrm{I}_{\{0\}}$). In our talk we discuss the rate of this convergence.

For a complete separable metric space (X,d) let $\mathcal{M}_1(X)$ denote the space of all Borel probability measures on X having a finite first moment endowed with the standard Wasserstein metric W_1. It is well known that $(\mathcal{M}_1(X), W_1)$ is also a complete separable metric space (see, for instance, [3, Chapter 6]).

Fix an arbitrary measure $\mu \in \mathcal{M}_1(\mathbb{R})$ and let λ and λ_0 denote the images of μ under the action of the random mappings $x(\cdot,1)\colon \mathbb{R} \to \mathbb{R}$ and $x_0(\cdot,1)\colon \mathbb{R} \to \mathbb{R}$ respectively. It can be easily checked that λ and λ_0 are random elements in the space $\mathcal{M}_1(\mathbb{R})$, and so we can consider their distributions Λ and Λ_0 in this space. Note that Λ and Λ_0 are elements of the space $\mathcal{M}_1(\mathcal{M}_1(\mathbb{R}))$.

Theorem. *Let $\{x(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ be a Harris flow with covariance function Γ, which has compact support, and $\{x_0(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ be the Arratia flow. Assume that*

$$\operatorname{supp} \mu \subset [0;1]$$

and

$$d(\Gamma) < \frac{1}{100}.$$

Then

$$W_1(\Lambda, \Lambda_0) \leqslant C \cdot d(\Gamma)^{1/22},$$

where the constant $C > 0$ does not depend on μ and Γ.

This talk is based on the results of joint work with Prof. A. A. Dorogovtsev, which are going to be published in [2].

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TEST FOR CHECKING HYPOTHESIS ON EXPECTATION AND COVARIANCE FUNCTION OF A RANDOM SEQUENCE

T. O. IANEVYCH AND YU. V. KOZACHENKO

Let us consider the stationary sequence $\{\gamma(n), n \geq 1\}$ for which $\mathsf{E}\gamma(n) = a$ is its expectation and $\mathsf{E}(\gamma(n) - a)(\gamma(n+m) - a) = B(m)$, $m \geq 0$ is its covariance function. Hereinafter we'll consider stationarity in a strict sense.

We assume that we have $N + M$ $(N, M > 0)$ consecutive observations of this random sequence. Let us consider the estimators for the expectation and covariance function as follows:

$$\widehat{a}_N(m) = \frac{1}{N}\sum_{n=1}^{N}\gamma(n+m), \quad 0 \leq m \leq M-1,$$

$$\widehat{B}_N(m) = \frac{1}{N}\sum_{n=1}^{N}(\gamma(n) - \widehat{a}_N(0))(\gamma(n+m) - \widehat{a}_N(m)).$$

For every estimator above we can evaluate the quantities $(0 \leq m \leq M-1)$

$$E_N^a(m) := \mathsf{E}(\widehat{a}_N(m) - a)^2 = \frac{1}{N^2}\sum_{n=1}^{N}\sum_{k=1}^{N}B(n-k) = r_N(0),; \qquad (1)$$

$$E_N^B(m) := \mathsf{E}\widehat{B}_N(m) = B(m) - \frac{1}{N^2}\sum_{n=1}^{N}\sum_{k=1}^{N}B(m-(n-k)) = \qquad (2)$$

$$= B(m) - r_N(m), \qquad (3)$$

So, the following random variables are square Gaussian (see, for example, [1]):

$$\xi_N^a(m) := (\widehat{a}_N(m) - a)^2 - E_N^a(m) = (\widehat{a}_N(m) - a)^2 - r_N(0), \qquad (4)$$

$$\xi_N^B(m) := \widehat{B}_N(m) - E_N^B(m) = \widehat{B}_N(m) - B(m) + r_N(m), \quad 0 \leq m \leq M-1. \qquad (5)$$

Let us define the vectors

$$\vec{\xi}_N(m)^T = (\xi_N^a(m), \xi_N^B(m)), \quad 0 \leq m \leq M-1. \qquad (6)$$

For any semi-definite matrix $\mathbf{B}_m = (b_{ij}(m))_{i,j=1,2}$ the random variables

$$\eta_N(m) := \vec{\xi}_N(m)^T \mathbf{B}_m \vec{\xi}_N(m) \tag{7}$$

are actually the quadratic forms of square Gaussian random variables.

Example. If $b_{ij} = \begin{cases} 1, & i = j; \\ 0, & i \neq j. \end{cases}$ (that is, $\mathbf{B}$ is the identity matrix of order 2), then $\eta_N(m) = (\xi_N^a(m))^2 + (\xi_N^B(m))^2$ and $\mathsf{E}\eta_N(m) = (\mathsf{E}\xi_N^a(m))^2 + (\mathsf{E}\xi_N^B(m))^2$.

If we consider the particular case when $\mathbf{B}_m = \mathbf{I}_2$ in the relations (1)-(7), then we can construct the following goodness-of-fit test.

Criterion. Let the null hypothesis H_0 state that for non-centered Gaussian stationary sequence $\{\gamma(n), n \geq 1\}$, the expectation $a = a_0$ and its covariance function $B(m) = B_0(m)$, $m \geq 0$. And the alternative H_a imply the opposite statement. The random variables $\eta_N(m)$ are as determined in (1)-(7) with $\mathbf{B}_m = \mathbf{I}_2$.

If for significance level α, some fixed $p \geq 1$ and $M < N$ $(M, N \in \mathbb{N})$

$$\left[\sum_{m=0}^{M-1} \left(\sqrt{\frac{\frac{(\widehat{a}_N(m)-a)^4}{2} + (\widehat{B}_N(m) - B(m))^2}{\mathsf{E}\eta_N(m)}}\right)^p\right]^{1/p} > \varepsilon_\alpha,$$

H_0 should be rejected and accepted otherwise.

Here ε_α is a critical value that can be found from the equation $U\left(\frac{\varepsilon_\alpha - \sqrt{2}A_N}{M^{1/p}}\right) = \alpha$, with $A_N := \left[\sum_{m=0}^{M-1}\left(\frac{(r_N(0))^2 + (r_N(m))^2}{\mathsf{E}\eta_N(m)}\right)^{p/2}\right]^{1/p}$ and taking into account the restriction $\varepsilon_\alpha > \sqrt{2}A_N + pM^{\frac{1}{p}}\left(1 + \sqrt{1 + \frac{2}{p}}\right)$.

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LOCAL UNIVERSALITY FOR REAL ROOTS OF RANDOM TRIGONOMETRIC POLYNOMIALS

A. IKSANOV AND A. MARYNYCH

We are interested in random trigonometric polynomials $X_n : \mathbb{R} \to \mathbb{R}$ of the form

$$X_n(t) = \sum_{k=1}^{n} (\xi_k \sin(kt) + \eta_k \cos(kt)),$$

where the coefficients $\xi_1, \eta_1, \xi_2, \eta_2, \ldots$ are real random variables. In a recent paper, Azaïs et al. [1] conjectured that if $\xi_1, \eta_1, \xi_2, \eta_2, \ldots$ are independent identically distributed (i.i.d.) with zero mean and finite variance, then the number of real zeros of X_n in the interval $[a/n, b/n]$ converges in distribution (without normalization) to the number of zeros in the interval $[a, b]$ of a stationary Gaussian process $Z := (Z(t))_{t\in\mathbb{R}}$ with zero mean and

$$\mathrm{Cov}(Z(t), Z(s)) = \mathrm{sinc}(t - s), \quad t, s \in \mathbb{R},$$

where

$$\mathrm{sinc}\, t = \begin{cases} (\sin t)/t, & \text{if } t \neq 0, \\ 1, & \text{if } t = 0. \end{cases}$$

The limit distribution does not depend on the distribution of ξ_1, a phenomenon referred to as *local universality*. Azaïs et al. [1] proved their conjecture assuming that ξ_1 has an infinitely smooth density that satisfies certain integrability conditions. However, as they remarked, even the case of the Rademacher distribution $\mathbb{P}\{\xi_1 = \pm 1\} = 1/2$ remained open. We prove the conjecture of [1] in full generality. Moreover, we also establish similar local universality results for the centered random vectors (ξ_k, η_k) having an arbitrary covariance matrix or belonging to the domain of attraction of a two-dimensional α-stable law.

The talk is based on a recent joint work [2] with Z. Kabluchko (Münster, Germany).

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LIMIT THEOREMS FOR MULTIDIMENSIONAL RENEWAL SETS

A. ILIENKO

We are interested in the geometry of renewal sets constructed from partial sums of i.i.d. random variables with multidimensional indices.

Let $\left(\xi_{\mathbf{m}}, \mathbf{m} \in \mathbb{N}^d\right)$ be a multi-indexed family of i.i.d. and a.s. non-negative random variables with finite mean $\mu > 0$. Denote by $S_{\mathbf{n}}$, $\mathbf{n} \in \mathbb{N}^d$, their partial sums:

$$S_{\mathbf{n}} = \sum_{\mathbf{m} \leq \mathbf{n}} \xi_{\mathbf{m}},$$

and by $\mathcal{M}_t$, $t > 0$, the corresponding renewal sets:

$$\mathcal{M}_t = \{\mathbf{n} \in \mathbb{N}^d \colon S_{\mathbf{n}} \leq t\}.$$

In the recent monograph by O. Klesov [1], a number of a.s. limit theorems for the renewal process $N(t) = \operatorname{card} \mathcal{M}_t$ are derived. So, these results answer the question of "how large" are $\mathcal{M}_t$. On the contrary, we study the location and the shape of $\mathcal{M}_t$.

Consider the "subhyperbolic" set

$$\mathcal{H} = \left\{\mathbf{x} \in \mathbb{R}_+^d \colon \prod_{i=1}^{d} x_i \leq \mu^{-1}\right\}.$$

We show that the rescaled sets $t^{-1/d}\mathcal{M}_t$ converge (in a sense to be specified) towards $\mathcal{H}$ as $t \to \infty$. The rate of convergence (in the form of the Marcinkiewicz-Zygmund strong law of large numbers) and the law of iterated logarithm are studied as well.

The talk is based on joint work with I. Molchanov (Bern, Switzerland).

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A NOTE ON THE KOLMOGOROV-MARCINKIEWICZ-ZYGMUND TYPE SLLN FOR ELEMENTS OF AUTOREGRESSION SEQUENCES

M. K. ILIENKO

In the paper [1], the Kolmogorov-Marcinkiewicz-Zygmund type SLLN for sums of i.i.d. random variables is studied in a way that authors obtain necessary and sufficient conditions providing almost sure convergence of the series $\sum_{n=1}^{\infty} \frac{|S_n|}{n^{1+1/p}}$.

Got interested in the subject we obtain results of a sort for sums whose terms are elements of random autoregression sequences by means of technique developed by V. Buldygin and M. Runovska, [2]. Thus, consider a zero-mean linear regression sequence of random variables $(\xi_k) = (\xi_k,\ k \geq 1)$:

$$\xi_1 = \eta_1, \quad \xi_k = \alpha_k \xi_{k-1} + \eta_k, \quad k \geq 2, \tag{1}$$

where (α_k) is a nonrandom real sequence, and (η_k) is a sequence of independent symmetric random variables. Set $S_n = \sum_{k=1}^{n} \xi_k$, $n \geq 1$, and study necessary and sufficient conditions providing almost sure convergence of the series

$$\sum_{n=1}^{\infty} \frac{S_n}{n^{1+\frac{1}{p}}}, \tag{2}$$

where $p > 0$. Let us formulate one of the results.

Theorem 1. *Let in* (1) $\alpha_k = \alpha = const$, *for any* $k \geq 2$, *and* (η_k) *be a sequence of independent copies of a symmetric random variable* η. *The series* (2) *converges a.s. iff one of the following two conditions is satisfied*:
a) $-1 \leq \alpha < 1$, $p > 0$, *and* $E|\eta|^p < \infty$;
b) $\alpha = 1$, $0 < p < 1$, *and* $E|\eta|^{\frac{p}{1-p}} < \infty$.

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THE DETECTION OF HIDDEN PERIODICITIES IN REGRESSION WITH LOCALLY TRANSFORMED NOISE

A. V. IVANOV

The talk is devoted to the solution to the problem of detecting hidden periodicities in discrete time regression model with nonlinearly locally transformed, possibly, strongly dependent Gaussian stationary time series in the capacity of random noise. The results obtained generalize the correspondent results of the paper [1] for time continuous observation model where one can find necessary references and discussion.

To prove asymptotic normality of the joint least squares estimate (l.s.e.) in Walker sense of amplitudes and angular frequencies of a sum of harmonic oscillations we study general nonlinear regression and obtain a general theorem on l.s.e. asymptotic normality using the CLT from the paper [2]. Then asymptotic normality of the l.s.e. in trigonometric regression follows from this theorem.

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ABSTRACT PRIME NUMBER THEOREMS FOR ADDITIVE ARITHMETICAL SEMIGROUPS

E. KAYA

Let (G, ∂) be an additive arithmetical semigroup. By definition, G is a free commutative semigroup with identity element 1, generated by a countable set P of primes and admitting an integer valued degree mapping $\partial : G \to \mathbb{N} \cup \{0\}$ with the properties

(i) $\partial(1) = 0$ and $\partial(p) > 0$ for all $p \in P$,
(ii) $\partial(ab) = \partial(a) + \partial(b)$ for all $a, b \in G$,
(iii) the total number $G(n)$ of elements $a \in G$ of degree $\partial(a) = n$ is finite for each $n \geq 0$.

Obviously, $G(0) = 1$ and G is countable. Let

$$\pi(n) := \#\{p \in P : \partial(p) = n\}$$

denote the total number of primes of degree n in G. The asymptotic behavior of $\pi(n), n \to \infty$, was called "abstract prime number theorem".

In this talk, we prove abstract prime number theorems for additive arithmetical semigroups.

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LAW OF LARGE NUMBERS FOR MARTINGALES

O. I. KLESOV

Let $\{S_n\}$ be a martingale with respect to the natural flow of σ-algebras. Chow [2] proved the following result.

Theorem 1. *Let $X_n = S_n - S_{n-1}$ and $\{C_k\}$ be a nonincreasing sequence positive numbers. For $\alpha \geq 1$ and $2\alpha \geq \beta > 0$, if there exists i_0 such that for $i \geq i_0$*

$$\mathbf{E}\left[|S_i|^{2\alpha}\right] \leq A\mathbf{E}\left[\left(\sum_{k=1}^{i} X_k^2\right)^{\alpha}\right], \tag{1}$$

$$i^{\alpha-1}C_i^{2\alpha-\beta} \leq A, \qquad \sum_{k=i}^{\infty} C_k^{2\alpha}k^{\alpha-2} \leq AC_i^{\beta} \tag{2}$$

where A is a constant, independent of i, and if

$$\sum_{k=1}^{\infty} C_k^{\beta}\mathbf{E}\left[|X_k|^{2\alpha}\right] < \infty, \tag{3}$$

then $\lim C_nS_n = 0$ almost surely.

Later Burkholder [1] proved that (1) is always satisfied. We are concerned with conditions (2) and (3) in the talk.

Some sufficient conditions for the strong law of large numbers for martingales with continuous time are discussed similarly to the case of discrete time (see [3]). A special attention is paid to the case of solutions of stochastic differential equations.

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STRONG RANDOM OPERATORS GENERATED BY STOCHASTIC FLOWS

I. A. KORENOVSKA

We consider a strong random operator (SRO) [1] T_t in $L_2(\mathbb{R})$ that describes shift of functions along Arratia flow [2] $\{x(u,t),\ u \in \mathbb{R},\ t \geq 0\}$, i.e. for fixed $t > 0$

$$(T_t f)(u) = f(x(u,t))$$

where $f \in L_2(\mathbb{R})$, $u \in \mathbb{R}$.

Lemma 1. *T_t is not a bounded SRO* [3] *in $L_2(\mathbb{R})$.*

The image of a compact set $K \subset L_2(\mathbb{R})$ under a SRO in general is not defined. For $T_t(K)$ to be well-defined and to be a random compact it is sufficient to show that T_t has a continuous modification on K.

Theorem 1. *Let Φ be a subset of Sobolev space $W_2^1(\mathbb{R})$ and*

$$\sup_{f,g \in \Phi} \int_{\mathbb{R}} (f'(u) - g'(u))^2 (|u|+1)^3 du < \infty.$$

Then T_t has a continuous modification on Φ.

Necessary and sufficient conditions under which T_t saves convergence are presented.

Theorem 2. *Let $\{f_n\}_{n=1}^{\infty} \subset L_2(\mathbb{R})$ be a sequence such that $f_n \to 0, n \to \infty$, in $L_2(\mathbb{R})$. If $P\{ \lim_{n\to\infty} \|T_t f_n\|_{L_2(\mathbb{R})} = 0 \} = 1$, then $f_n \to 0, n \to \infty$, a.e. under Lebesgue measure λ on $\mathbb{R}$.*

Theorem 3. *Let $\{f_n\}_{n=1}^{\infty} \subset L_2(\mathbb{R})$ be a sequence for which the following conditions hold*

(1) $f_n \xrightarrow[n\to\infty]{L_2(\mathbb{R})} 0$;
(2) $f_n \to 0, n \to \infty$, *a.e. under Lebesgue measure λ on $\mathbb{R}$* ;
(3) $\exists C > 0 :\ \ \forall n \geq 1 \quad supp f_n \subset [-C; C]$.

Then $P\{ \lim_{n\to\infty} \|T_t f_n\|_{L_2(\mathbb{R})} = 0 \} = 1$.

Convergence almost everywhere isn't sufficient to save the convergence under T_t.

Example 1. Let fix $t > 0$ and consider sequences $\{c_n\}_{n=0}^{\infty}, \{a_n\}_{n=0}^{\infty}$ such that $c_0 = 1,\ a_0 = 0$ and

$$c_n = (\ln n)^{\frac{1}{4}}, \quad a_n = a_{n-1} + 1 + 2n(t \ln 2)^{\frac{1}{2}} \quad \forall n \in \mathbb{N}.$$

Then for every $\delta \in (0; 2\sqrt{t})$ and function $f_n = \frac{1}{c_n} \mathbb{1}_{(a_n; a_{n+1}]}$ the following statement holds

$$P\{\ \overline{\lim}_{n\to\infty} \|T_t f_n\|^2_{L_2(\mathbb{R})} \geq \delta\ \} = 1.$$

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HYPOTHESIS TESTING OF THE DRIFT PARAMETER SIGN FOR FRACTIONAL ORNSTEIN–UHLENBECK PROCESS

A. KUKUSH, Y. MISHURA, AND K. RALCHENKO

We consider the Langevin equation $dX_t = \theta X_t dt + dB_t^H$, which contains an unknown drift parameter θ, and where the noise is modeled as fractional Brownian motion with known Hurst index H. The solution corresponds to the fractional Ornstein–Uhlenbeck process. We propose comparatively simple test for testing the null hypothesis $H_0 \colon \theta \le 0$ against the alternative $H_1 \colon \theta > 0$ and prove its consistency. Contrary to the previous works ([1, 2, 3]), our approach is applicable for all $H \in (0,1)$. The test is based on the observations of the process X at two points: 0 and T. The distribution of the test statistic is computed explicitly, and the power of test can be found numerically for any given simple alternative. Also we consider the hypothesis testing $H_0 \colon \theta \ge \theta_0$ against $H_1 \colon \theta \le 0$, where $\theta_0 \in (0,1)$ is some fixed number.

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GOODNESS-OF-FIT TEST IN A MULTIVARIATE ERRORS-IN-VARIABLES MODEL

A. G. KUKUSH AND YA. V. TSAREGORODTSEV

A homoscedastic multivariable functional errors-in-variables model $AX \approx B$ is considered, where the data matrices A and B are observed with additive errors and a matrix parameter X is to be estimated. The errors are uncorrelated, with unknown variance and vanishing third moments, and error distribution in a row of data matrix $[AB]$ does not depend on the row. The number m of rows in A and B is increasing while the size of X is fixed. A goodness-of-fit test is constructed based on the total least squares (TLS) estimator. Under null hypothesis, the proposed test is asymptotically chi-squared, with certain number of degrees of freedom. Local alternatives are introduced, where the output matrix B is disturbed with nonlinear term proportional to $m^{-\frac{1}{2}}$. Under the local alternatives, the test statistic has asymptotic noncentral chi-squared distribution, with the same number of degrees of freedom. The larger the noncentrality parameter the larger power of the test. In the presence of intercept term, the test fails and has to be modified.

The condition about uncorrelated errors with equal variances can be violated, moreover some columns of data matrix can be free of errors. For such heteroscedastic model, the elementwise-weighted total least squares (EWTLS) estimator replaces the TLS estimator. The EWTLS estimator is consistent [1], its asymptotic normality can be shown like in [2], and the corresponding goodness-of-fit test can be elaborated.

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FRACTIONAL POISSON FIELDS AND MARTINGALES

N. LEONENKO

We present new properties for the Fractional Poisson process [2,3], Fractional non-homogeneous Poisson process [6] and the Fractional Poisson fields on the plane [4]. A martingale characterization for Fractional Poisson processes is given. We extend this result to Fractional Poisson fields, obtaining some other characterizations. The fractional differential equations are studied. The covariance structure is given. Finally, we give some simulations of the Fractional Poisson fields on the plane.

This is a joint work with G. Aletti (University of Milan, Italy) and E. Merzbach (Bar Ilan University, Israel).

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THE AUTO- AND CROSS-DISTANCE CORRELATION FUNCTIONS OF A MULTIVARIATE TIME SERIES AND THEIR SAMPLE VERSIONS

T. MIKOSCH

This is joint work with R.A. Davis, P. Wan (Columbia Statistics), and M. Matsui (Nagoya). Feuerverger (1993) and Székely, Rizzo and Bakirov (2007) introduced the notion of distance covariance/correlation as a measure of independence/dependence between two vectors of arbitrary dimension and provided limit theory for the sample versions based on an iid sequence. The main idea is to use characteristic functions to test for independence between vectors, using the standard property that the characteristic function of two independent vectors factorizes. Distance covariance is a weighted version of the squared distance between the joint characteristic function of the vectors and the product of their marginal characteristic functions. Similar ideas have been used in the literature for various purposes: goodnes-of-fit tests, change point detection, testing for independence of variables,... ; see work by Meintanis, Huškova, and many others. In contrast to Székely et al. who use a weight function which is infinite on the axes, the latter authors choose probability density weights. Z. Zhou (2012) extended distance correlation to time series models for testing dependence/independence in a time series at a given lag. He assumed a "physical dependence measure".

In our work we consider the distance covariance/correlation for general weight measures, finite or infinite on the axes or at the origin. These include the choice of Székely et al., probability and various Lévy measures. The sample versions of distance covariance/correlation are obtained by replacing the characteristic functions by their sample versions. We show consistency under ergodicity and weak convergence to an unfamiliar limit distribution of the scaled auto- and cross-distance covariance/correlation functions under strong mixing. We also study the auto-distance correlation function of the residual process of an autoregressive process. The limit theory is distinct from the corresponding theory of an iid noise process. We illustrate the theory for simulated and real data examples.

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CONVEX HULLS OF LÉVY PROCESSES

I. MOLCHANOV AND F. WESPI

A number of existing results concern convex hulls of stochastic processes, especially the Brownian motion and random walks, see [1, 5]. In contrary, considerably less is known about convex hulls of general Lévy processes with the exception of some results for symmetric α-stable Lévy processes in $\mathbb{R}^d$ with $\alpha \in (1, 2]$, see [2].

Let X_t, $t \geq 0$, be a Lévy process with values in $\mathbb{R}^d$. We study properties of the random compact set

$$Z_s = \overline{\mathrm{co}}\{X_t, 0 \leq t \leq s\}$$

which is the closed convex hull of the path of the process up to time t.

The most important geometric functionals of a convex body K in $\mathbb{R}^d$ are intrinsic volumes $V_j(K)$, $j = 0, 1, \ldots, d$, see [4]. In dimension $d = 2$, $V_2(K)$ is the area and $2V_1(K)$ is the perimeter.

We obtain a criterion for the existence of the moments of $V_j(Z_s)$ that generalise the result of [2] for Lévy processes with independent coordinates. Using approximation with random walks and the results of [5], we obtain an explicit formula for all expected intrinsic volumes if X_t is a symmetric stable Lévy process generalising this result for the Brownian motion from [1].

It is shown that X_s almost surely belongs to the interior of Z_s for all s. Finally it is shown that the normalised convex hulls converge to the convex hulls of a symmetric stable Lévy process.

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PROBABILITIES OF LARGE DEVIATIONS OF RANDOM NOISE COVARIANCE FUNCTION ESTIMATOR IN NONLINEAR REGRESSION MODEL

K. K. MOSKVYCHOVA

Consider a model of observations $X(t) = g(t,\theta) + \xi(t),\ t \in [0,\infty)$, where $g : [0,\infty) \times \Theta \to \mathbb{R}$ is continuous function depending on unknown parameter $\theta = (\theta_1, ..., \theta_q) \in \Theta \subset \mathbb{R}^q$, Θ is bounded open convex set, $\xi = \{\xi(t), t \in \mathbb{R}\}$ is a real mean square and almost sure continuous stationary Gaussian process with zero mean and positive bounded spectral density $f = \{f(\lambda),\ \lambda \in \mathbb{R}\}$.

Any random vector $\widehat{\theta}_T = (\widehat{\theta}_{1T}, \dots \widehat{\theta}_{qT}) \in \Theta$ satisfying relation $Q_T(\widehat{\theta}_T) = \min\limits_{\tau\in\Theta} \int\limits_0^T [X(t) - g(t,\tau)]^2 dt$ is said to be the least squares estimate of unknown parameter $\theta \in \Theta$ on observation interval [0,T].

As estimator of covariance function $B(t),\ t \in \mathbb{R}$ of the process ξ we choose correlogram $B_T(z, \widehat{\theta}_T) = T^{-1} \int\limits_0^T (X(t+z) - g(t+z, \widehat{\theta}_T))(X(t) - g(t, \widehat{\theta}_T))dt$, $z \in [0,H]$. We find sufficient conditions under which exist constants A_0 and b_0 such that for $T > T_0$, $R > R_0$

$$P\left\{T^{1/2} \sup_{z\in[0,H]} |B_T(z,\hat{\theta}_T) - B(z)| \geq R\right\} \leq A_0 \exp\{-b_0 R\} \left(1 + \gamma(T,R)\right),$$

where $\lim\limits_{T\to\infty, R\to\infty} \gamma(T,R) = 0$.

In particular, the result significantly sharpens the results obtained in [1].

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ASYMPTOTIC NORMALITY OF REGRESSION PARAMETER ESTIMATOR IN THE CASE OF LONG-RANGE DEPENDENT REGRESSORS AND NOISE WITH SEASONAL EFFECTS

I. V. ORLOVSKYI

Consider a regression model

$$X_j = \sum_{i=1}^{q} \theta_i z_{ij} + \varepsilon_j,\ z_{ij} = a_{ij} + y_{ij},\ j = \overline{1,N}, \tag{1}$$

where $\theta = (\theta_1, ..., \theta_q) \in \mathbb{R}^q$ is a vector of unknown parameters, $\{a_{ij},\ j \in \mathbb{N}\} \subset \mathbb{R},\ i = \overline{1,q}$, are some non random sequences, $y_{ij},\ j \in \mathbb{Z},\ i = \overline{1,q}$, and $\varepsilon_j,\ j \in \mathbb{Z}$, are independent real stationary Gaussian sequences with zero means that satisfy long-range dependence condition with seasonal effects i.e. their covariance functions have the form $B_i(n) = Ey_{in}y_{i0} = \cos\chi_i n \cdot L_i(|n|)|n|^{-\alpha_i}$, $B(n) = E\varepsilon_n\varepsilon_0 = \cos\chi_0 n \cdot L_0(|n|)|n|^{-\alpha_0}$, $n \in \mathbb{Z}$, where $L_i(t),\ t > 0$, are slowly varying at infinity functions, $B_i(0) = \sigma_i^2 > 0$, $\alpha_i \in \left(\frac{1}{2}, 1\right)$, $\chi_i \in [0, \pi)$, $i = \overline{0,q}$.

Definition 1. Least squares estimator of unknown parameter θ obtained from observations $\left\{X_j, z_{ij},\ i = \overline{1,q},\ j = \overline{1,N}\right\}$ of the type (1) is said to be any random vector $\widehat{\theta}_N = \widehat{\theta}_N\left(X_j, z_{ij},\ i = \overline{1,q},\ j = \overline{1,N}\right)$ having property $S_N(\widehat{\theta}_N) = \inf\limits_{\tau \in \mathbb{R}^q} S_N(\tau),\ S_N(\tau) = \sum\limits_{j=1}^{N}\left[X_j - \sum\limits_{i=1}^{q}\tau_i z_{ij}\right]^2.$

Sufficient conditions of asymptotic normality of least squares estimator of unknown parameter θ of model (1) are presented in the talk. Statements obtained generalize some results derived in the book [1].

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ON SOME MARKOV PROCESSES RELATED TO A SYMMETRIC STABLE PROCESS

M. M. OSYPCHUK AND M. I. PORTENKO

For fixed parameters $c > 0$ and $\alpha \in (1, 2]$ we put

$$g(t, x, y) = \frac{1}{\pi} \int_0^\infty e^{-ct\xi^\alpha} \cos \xi(y - x)\, d\xi, \quad t > 0,\ x \in \mathbb{R},\ y \in \mathbb{R}. \tag{1}$$

There exists a standard Markov process $(x(t), \mathbb{P}_x)$ on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ (in Dynkin's sense) such that

$$\mathbb{P}_x(\{x(t) \in \Gamma\}) = \int_\Gamma g(t, x, y)\, dy, \quad t > 0,\ x \in \mathbb{R},\ \Gamma \in \mathcal{B}(\mathbb{R})$$

Introduce the following stopping times

$$\tau^0 = \inf\{s \geq 0 : x(s) = 0\} \quad \text{and} \quad \sigma = \inf\{s \geq 0 : x(s)x(0) \leq 0\}$$

and consider the function

$$g^*(t, x, y) = g(t, x, y) - g(t, -|x|, |y|)$$

defined for $t > 0$, $x \in \mathbb{R}_0$ and $y \in \mathbb{R}_0$ ($\mathbb{R}_0 = \mathbb{R} \setminus \{0\}$).

If $\alpha = 2$, then $\mathbb{P}_x(\{\tau^0 = \sigma\}) = 1$ for all $x \in \mathbb{R}$ and

$$\mathbb{P}_x(\{x(t) \in \Gamma\} \cap \{\tau_0 > t\}) = \int_\Gamma g^*(t, x, y)\, dy \tag{2}$$

for $t > 0$, $x \in \mathbb{R}_0$, $\Gamma \in \mathcal{B}(\mathbb{R}_0)$.

If $1 < \alpha < 2$, then $\mathbb{P}_x(\{\tau^0 > \sigma\}) = 1$ for $x \in \mathbb{R}_0$ and (2) is not true.

Denote by $(x^0(t), \mathbb{P}_x^0)$ and $(x^*(t), \mathbb{P}_x^*)$ the Markov processes on $(\mathbb{R}_0, \mathcal{B}(\mathbb{R}_0))$ whose transition probabilities are given, respectively, by the left-hand and right-hand sides of (2). We investigate some properties of these processes, in particular, we find out their potential operators, the distribution functions of τ^0 and τ^* (this is the life time of the process $(x^*(t))_{t\geq 0}$) and show that the distribution functions of τ^* and σ are different. In the case of $\alpha = 2$ the functions g and g^* turn out to be connected by the Feynman-Kac transformation in some weak sense and they do not in the case of $1 < \alpha < 2$. In the latter one, the process $(x^*(t))_{t\geq 0}$ is shown to be a solution to a martingale problem.

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INTEGRAL REPRESENTATIONS OF KARAMATA'S TYPE FOR ORV FUNCTIONS WITH NONDEGENERATE GROUPS OF REGULAR POINTS

V. V. PAVLENKOV

Let $\mathbb{FM}_+$ be the set of positive and measurable functions $f = (f(x), x \geq A)$ for some $A > 0$. For given $f \in \mathbb{FM}_+$ introduce the limit functions

$$f^*(\lambda) = \limsup_{x\to\infty} \frac{f(\lambda x)}{f(x)} \quad \text{and} \quad f_*(\lambda) = \liminf_{x\to\infty} \frac{f(\lambda x)}{f(x)}, \quad \lambda > 0,$$

witch take values in $[0, \infty]$.

A number $\lambda > 0$ is called *a regular point* of the function f, denoted $\lambda \in \mathbb{G}_r(f)$, if

$$f_*(\lambda) = f^*(\lambda) \in (0, \infty).$$

If $\mathbb{G}_r(f) = \mathbf{R}_+$, then f is called *regularly varying* (RV) function. If $f^*(\lambda) < \infty, \lambda > 0$, then f is called *O-regularly varying* (ORV) function.

The set $\mathbb{G}_r(f)$ of regular points of f is a multiplicative subgroup of $\mathbf{R}_+$ with $1 \in \mathbb{G}_r(f)$. If $\mathbb{G}_r(f) = \{1\}$, then $\mathbb{G}_r(f)$ is called *degenerate*, otherwise *nondegenerate.* ORV functions with nondegenerate group of regular points were introduced and studied by Buldygin, Klesov and Steinebach.

Karamata's theorem on the integral representation of RV function is well known. The similar result for ORV function is also known. Integral representations of Karamata's type for ORV functions with nondegenerate groups of regular points are considered in the talk.

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ON A SELECTION PROBLEM FOR SMALL NOISE PERTURBATION OF NON-LIPSCHITZ O.D.E.

A. YU. PILIPENKO

Consider an equation

$$X^\varepsilon(t) = x + \int_0^t a(s, X^\varepsilon(s))ds + \varepsilon B(t),$$

where $B(t), t \geq 0$ is a multidimensional Wiener process, the function $a = a(t, x) : [0, \infty) \times R^d$ satisfies Lipschitz condition in x everywhere except of the hyperplane $R^{d-1} \times \{0\}$.

We identify limits of $\{X^\varepsilon\}$ distributions as $\varepsilon \to 0$. It appears that the behavior of the limit process depends on signs of the normal component of the drift at the upper and lower half-spaces in a neighborhood of the hyperplane, all cases are considered.

We also discuss limits of the sequence ($d = 1$)

$$Y^\varepsilon(t) = \int_0^t (c_+ 1_{Y^\varepsilon(s)>0} - c_- 1_{Y^\varepsilon(s)<0})|Y^\varepsilon(s)|^\beta ds + \varepsilon B_\alpha(t),\ t \geq 0,$$

where $\beta < 1$, $c_\pm > 0$, B_α is an α self-similar process.

It appears that the limit of $\{Y^\varepsilon\}$ as $\varepsilon \to 0$ is closely related with the limit behavior of a solution to the equation

$$Y(t) = \int_0^t (c_+ 1_{Y(s)>0} - c_- 1_{Y(s)<0})|Y(s)|^\beta ds + B_\alpha(t),\ t \geq 0,$$

as $t \to \infty$.

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REPRESENTATION OF GAUSSIAN FIELD BY CHENTSOV RANDOM FIELD

N. V. PROKHORENKO

Let us consider Gaussian field with zero expectation and following covariance function:

$$R(\bar{s},\bar{t}) = \prod_{i=1}^{n} u_i(\min\{s_i,t_i\})v_i(\max\{s_i,t_i\}), \tag{1}$$

$\bar{s} = (s_1,\ldots,s_n)$, $\bar{t} = (t_1,\ldots,t_n)$.

We will find the representation of this fields via Chentsov random field. The received outcomes can be used for research of functionals of the Gaussian fields. For example, to find the probability that Gaussian field crossing certain surface.

Theorem 1. *Let $Y(\bar{t})$, $\bar{t} = (t_1,\ldots,t_n)$, be any Gaussian field with $E[Y(\bar{t})] = 0$ and covariance function (1) for all $\bar{s},\bar{t} \in [0,\infty)^n$. Let the functions $\frac{u_i}{v_i}$, $i = \overline{1,k}$, and $\frac{v_i}{u_i}$, $i = \overline{k{+}1,n}$, have inverse functions $a_i = \left(\frac{u_i}{v_i}\right)^{-1}$ and $b_i = \left(\frac{v_i}{u_i}\right)^{-1}$ respectively.*

Assume that $\varphi_i(t_i), i = \overline{1,k}$, are continuous and strictly increasing functions, $\varphi_i(t_i), i = \overline{k{+}1,n}$, are continuous and strictly decreasing functions, $\eta_i(t_i), i = \overline{1,n}$, are continuous functions. Gaussian field $\frac{Y(\varphi(\bar{t}))}{\eta(\bar{t})}$, where $\varphi(\bar{t}) = (\varphi_1(t_1),\ldots,\varphi_n(t_n))$, $\eta(\bar{t}) = \prod_{i=1}^{n}\eta_i(t_i)$, and Chentsov random field are stochastically equivalent if and only if:

$$\varphi_i(t_i) = a_i(c_i^2 t_i), \quad \eta_i(t_i) = c_i v_i\left(a_i(c_i^2 t_i)\right), \quad i = \overline{1,k},$$

$$\varphi_j(t_j) = b_j(c_j^2 t_j), \quad \eta_j(t_j) = c_j u_j\left(b_j(c_j^2 t_j)\right), \quad j = \overline{k{+}1,n},$$

where $\bar{c} = (c_1,\ldots,c_n) \neq 0$.

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SMALL RANDOM PERTURBATIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH POWER COEFFICIENTS

YU. E. PRYKHODKO

Bafico and Baldi [1] considered differential equation

$$dX(t) = a(X(t))\,dt,$$

where coefficient a does not satisfy the Lipschitz condition at 0. They suggested to consider small random perturbations of this equations, i.e. stochastic differential equation

$$dX_\varepsilon(t) = a(X_\varepsilon(t))\,dt + \varepsilon\,dW(t),$$

for which existence and uniqueness of solutions is known; and studied the limit behavior of X_ε as $\varepsilon \to 0$.

We will generalize the result of [1] to the case of SDE

$$dX_\varepsilon(t) = a(X_\varepsilon(t))\,dt + \big(\sigma(X_\varepsilon(t)) + \varepsilon\big)\,dW(t)$$

with power coefficients $a(x) = a_\pm|x|^\alpha \operatorname{sign} x$ and $\sigma(x) = b_\pm|x|^\beta$, where $a_\pm > 0$, $b_\pm \geq 0$, $\alpha, \beta > 0$. To study the limit behavior of X_ε as $\varepsilon \to 0$ one can use the general methods developed in [2] and [3].

It is then proved [4] that the sequence of processes $\{X_\varepsilon(\cdot)\}$ is weakly convergent as $\varepsilon \to 0$ in the space of continuous functions.

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KANTOROVICH-RUBINSTEIN DISTANCE ON THE ABSTRACT WIENER SPACE

G. V. RIABOV

Let $(X, \|\cdot\|)$ be a separable Banach space equipped with a centered Gaussian measure μ. There exists a unique separable Hilbert space $(H, |\cdot|)$, continuously embedded into X, such that [1]

$$\int_X e^{il(x)}\mu(dx) = e^{-\frac{1}{2}|l|^2},\ l \in X^*.$$

The space $\mathcal{M}(X)$ of all Borel probability measures on X is equipped with the Kantorovich-Rubinstein distance:

$$W(\nu_1, \nu_2) = \inf_{\kappa} \int_X \int_X \|x_1 - x_2\| \kappa(dx_1, dx_2),$$

where the infimum is taken over all measures κ on X^2 with marginals ν_1 and ν_2. In the main theorem a representation of the distance W in terms of stochastic integral operator I ([1]) is given.

Theorem 1. *Let $\nu_1, \nu_2 \in \mathcal{M}(X)$ be such that*

$$\nu_2 - \nu_1 \ll \mu,\ \frac{d(\nu_2 - \nu_1)}{d\mu} \in L^2(X, \mu).$$

Then

$$W(\nu_1, \nu_2) = \inf_{u: Iu = \frac{d(\nu_2 - \nu_1)}{d\mu}} \int_X |u(x)|\mu(dx).$$

This theorem generalizes the result of [2], where an upper bound on $W(\nu, \mu)$ was obtained.

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ASYMPTOTIC NORMALITY OF THE LEAST MODULE ESTIMATOR IN REGRESSION MODEL WITH STRONGLY DEPENDENT RANDOM NOISE

I. N. SAVYCH

Consider a regression model $X(t) = g(t,\theta) + \xi(t),\ t \geq 0$, where $g(t,\theta) \in C\left([0,+\infty) \times \Theta^c\right)$, $\Theta \subset \mathbb{R}^q$, is an open bounded set, $\xi(t),\ t \in \mathbb{R}$, is a measurable stationary Gaussian process, $\mathsf{E}\,\xi(0) = 0$, $\mathsf{E}\,\xi(t)\xi(0) = \sum_{j=0}^{r} A_j B_{\alpha_j,\chi_j}(t),\ t \in \mathbb{R},\ r > 0$, where $A_j > 0$, $\sum_{j=1}^{r} A_j = 1$, $B_{\alpha_j,\chi_j}(t) = \frac{\cos(\chi_j t)}{(1+t^2)^{\frac{\alpha_j}{2}}}$, $j = \overline{0,r}$, $0 = \chi_0 < \chi_1 < ... < \chi_r < +\infty$, $\alpha_j \in (0,1)$. Let $\Phi(x) = \int_{-\infty}^{x} \varphi(t)dt$ be a distribution function of $\xi(0)$, where $\varphi(t) = e^{-\frac{t^2}{2}}/\sqrt{2\pi}, t \in \mathbb{R}$.

The least module estimator of parameter $\theta \in \Theta$ is any random vector $\hat{\theta}_T = \hat{\theta}_T\left(X(t),\ t \in [0,T]\right) \in \Theta^c$ for which $Q_T(\hat{\theta}_T) = \min_{\tau\in\Theta^c} Q_T(\tau)$, $Q_T(\tau) = \frac{1}{2}\int_0^T |X(t) - g(t,\tau)|\,dt$.

Let μ be spectral measure of regression function $g(t,\theta)$, spectral density $f(\lambda), \lambda \in \mathbb{R}$, of $\xi(t)$ is μ-admissible.

Denote by $d_{iT}^2(\theta) = \int_0^T \left(g'_{\theta_i}(t,\theta)\right)^2 dt, \quad d_T^2(\theta) = diag\left(d_{iT}^2(\theta)\right)_{i=1}^{q}.$

The normed estimator $d_T(\theta)(\widehat{\theta}_T - \theta)$ is asymptotically normal [1, 2] $N(0,\Gamma), \Gamma = 2\pi\Lambda\left[\int_{\mathbb{R}}\left(f(\lambda) + \Sigma_{n=1}^{\infty}\frac{(2n-1)!!}{(2n+1)(2n)!!}f^{*(2n+1)}(\lambda)\right)\mu(d\lambda,\theta)\right]\Lambda$, where $\Lambda = \left(\int_{\mathbb{R}}\mu(d\lambda,\theta)\right)^{-1}$, $f^{*j}(\lambda)$ is the $j-$th convolution of spectral density $f(\lambda)$.

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STRONG LAWS FOR ARRAYS OF INDEPENDENT RANDOM VARIABLES

U. STADTMÜLLER AND A. GUT

The present talk is devoted to complete convergence and the strong law of large numbers under moment conditions near those of the law of single logarithm (LSL) for i.i.d. arrays, where some recent papers [4, 3] triggered our investigations. More precisely, we wish to investigate possible limit theorems under moment conditions which are stronger than $2p$ for any $p < 2$, in which case we know that there is a.s. convergence to 0, and weaker than $E\,X^4/(\log^+|X|)^2 < \infty$ when a law of single logarithm holds. Furthermore we will discuss some special cases of weighted sums $\sum_{k=0}^{n} a_{nk} X_k$, in particular, Cesáro means of small order $0 < \alpha \leq 1$ where $a_{nk} = \binom{n-k+\alpha-1}{n-k} / \binom{n+\alpha}{n}$. Here a LIL for $1/2 < \alpha \leq 1$ and a LSL for $\alpha = 1/2$ as well as certain strong laws under appropriate moment conditions will be presented

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FOURIER SERIES AND FOURIER TRANSFORM OF GENERAL STOCHASTIC MEASURES

N. O. STEFANSKA

The Fourier series and the Fourier transform of general stochastic measures are considered in this work. The convergence of the Fourier series and the absolute continuity of stochastic measures are studied. The inversion theorem for Fourier transform and connection with convergence of stochastic integrals are established.

Let $L_0 = L_0(\Omega, \mathcal{F}, \mathsf{P})$ be the set of all real-valued random variables defined on the complete probability space $(\Omega, \mathcal{F}, \mathsf{P})$ (more precisely, the set of equivalence classes). Convergence in L_0 means the convergence in probability. Let X be an arbitrary set and $\mathcal{B}$ — a σ-algebra of subsets of X. Let μ be a stochastic measure (SM) on $\mathcal{B}$ (σ-additive mapping $\mu: \ \mathcal{B} \to L_0$).

An example of stochastic measures is $\mu(A) = \int_0^T \mathbf{1}_A(x)\, dX(x)$, where $X(x),\ 0 \le x \le T$, be a square integrable martingale or a fractional Brownian motion with Hurst index $H > 1/2$, $\mathcal{B}$ be the Borel σ-algebra on $[0, T]$.

Integrals of deterministic functions with respect to general SMs are well studied in [1](see also [2]). In particular, every bounded measurable function is integrable with respect to any μ. An analogue of the Lebesgue dominated convergence theorem holds for this integral (see [2, Proposition 7.1.1], [1, Corollary 1.2]).

The integral of a random function with respect to Lebesgue measure dt is considered in Riemann sense, this integral is studied in [3].

Let $\mathcal{B}$ be the Borel σ-algebra on $(-1/2, 1/2]$. For general SM μ on $\mathcal{B}$ we define the *Fourier series*

$$\mu \sim \sum_{k\in\mathbb{Z}} \xi_k \exp\{2\pi i k t\}, \text{ where } \xi_k = \int_{(-1/2,\ 1/2]} \exp\{-2\pi i k t\}\, d\mu(t).$$

We will use the notation

$$s_{m,n}(t) = \sum_{-m\le k\le n} \xi_k \exp\{2\pi i k t\}.$$

Theorem 1. *If all $\xi_k = 0$ a.s., $k \in \mathbb{Z}$, then $\mu(A) = 0$ a.s. for all $A \in \mathcal{B}$.*

We have the weak convergence of partial sums $s_{m,n}$ in the following sense.

Theorem 2. *Let function* $g : [-1/2,\ 1/2] \to \mathbb{R}$*,* $g(-1/2) = g(1/2)$*, satisfies Dini's condition uniformly on* $[-1/2,\ 1/2]$ *(see* [4, 14.35]*). Then*

$$\int\limits_{(-1/2,\ 1/2]} g(t)s_{m,n}(t)\,dt \xrightarrow{\mathsf{P}} \int\limits_{(-1/2,\ 1/2]} g(x)\,d\mu(x), \quad m,\ n \to \infty. \quad (1)$$

Corollary 1. *Let function* $g : [-1/2,\ 1/2] \to \mathbb{R}$ *satisfies the Hölder condition uniformly:*

$$|g(t+h) - g(t)| \le C|h|^{\alpha},$$

C, α *are independent of* t*,* $g(-1/2) = g(1/2)$*. Then* (1) *holds.*

Assumption 1. *For some* $p \ge 1$*, for any sequence* $f_n \in L_p\left((-1/2,\ 1/2]\,, dt\right)$ *such that* $\int\limits_{(-1/2,\ 1/2]} |f_n(x)|^p\,dx \to 0$*,* $n \to \infty$*, holds* $\int\limits_{(-1/2,\ 1/2]} f_n(x)\,d\mu(x) \xrightarrow{\mathsf{P}} 0$*,* $n \to \infty$*.*

The following theorem gives the absolute continuity of SM with respect to Lebesgue measure.

Theorem 3. *Let Assumption 1 holds,* $\sum_k |\xi_k| < +\infty$ *a.s.,* $\xi(t) = \sum_{k\in\mathbb{Z}} \xi_k \exp\{2\pi i k t\}$*, function* $g : [-1/2,\ 1/2] \to \mathbb{R}$*, satisfies Dini's condition uniformly on* $[-1/2,\ 1/2]$*. Then*

$$\int\limits_{(-1/2,\ 1/2]} g(t)\,d\mu = \int\limits_{(-1/2,\ 1/2]} \xi(t)g(t)\,dt.$$

In the sequel, $\mathcal{B}$ is the Borel σ-algebra in $\mathbb{R}^d$. We consider the *Fourier transform* of general SM μ on $\mathcal{B}$,

$$\hat{\mu}(t) = \int_{\mathbb{R}^d} e^{-2\pi i\langle t,x\rangle}\,d\mu(x) =$$
$$\int_{\mathbb{R}^d} \cos 2\pi\langle t,x\rangle\,d\mu(x) - i\int_{\mathbb{R}^d} \sin 2\pi\langle t,x\rangle\,d\mu(x), \quad t \in \mathbb{R}^d,$$

where $\langle t, x\rangle = \Sigma_{1\le k\le d} t_k x_k$. (See more details in [5]). By $\mathcal{D}$ we denote the set of infinitely differentiable functions $\varphi : \mathbb{R}^d \to \mathbb{C}$ with bounded support, $\mathbb{C}_0$ denotes the set of continuous functions $f : \mathbb{R}^d \to \mathbb{C}$ such that $\lim_{|t|\to\infty} f(t) = 0$.

We have the inversion theorem for Fourier transform.

Theorem 4. *For each function $\varphi \in \mathcal{D}$ holds*

$$\int_{\mathbb{R}^d} \varphi(x)\, d\mu(x) = \lim_{\alpha \to 0+} \int_{\mathbb{R}^d} \varphi(x)\, dx \int_{\mathbb{R}^d} e^{-4\pi^2 \alpha |t|^2} e^{2\pi i \langle x,t\rangle} \hat{\mu}(t)\, dt.$$

The following statement shows a connection with convergence of stochastic integrals.

Theorem 5. *Let μ_n and μ are SMs on $\mathcal{B}$, values $\mu_n(A)$, $A \in \mathcal{B}$, $n \geq 1$, are bounded by probability. Then the following statements are equivalent:*

1) for each $f \in \mathbb{C}_0$

$$\int_{\mathbb{R}^d} f\, d\mu_n \xrightarrow{\mathsf{P}} \int_{\mathbb{R}^d} f\, d\mu, \ n \to \infty;$$

2) for each $f \in \mathbb{C}_0 \cap L_1\left(\mathbb{R}^d, dt\right)$

$$\int_{\mathbb{R}^d} f(t)\hat{\mu}_n(t)\, dt \xrightarrow{\mathsf{P}} \int_{\mathbb{R}^d} f(t)\hat{\mu}(t)\, dt, \ n \to \infty.$$

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ON THE MONITORING OF CAPM PORTFOLIO BETAS

J. G. STEINEBACH

Despite substantial criticism, variants of the Capital Asset Pricing Model (CAPM) remain still the primary statistical tools for portfolio managers to assess the performance of financial assets. In the CAPM the risk of an asset is expressed through its correlation with the market, widely known as the beta. There is now a general consensus among economists that these portfolio betas are time-varying and that, consequently, any appropriate analysis has to take this variability into account. Moreover, recent advances in data acquisition and processing techniques have led to an increased research output concerning high-frequency models.

Within this framework, we first briefly discuss a modified functional CAPM, introduced in Aue et al. [1], that incorporates microstructure noise, as well as sequential monitoring procedures to test for the constancy of the portfolio betas in this setting. The main results provide some large-sample properties of these procedures.

In a second part of our talk, we present some more recent results of Chochola et al. [2], [3] on robust procedures for the monitoring of CAPM portfolio betas. Some asymptotic sequential tests are discussed for the (ordinary) CAPM in discrete time as well as some extensions of the procedures to a functional version of the CAPM are provided.

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APPLYING THE GENERALIZED SILVERMAN–SASS MATRIXES IN A SUMMATION OF A RANDOM SERIES

O. P. STRAKH

We consider a problem of a summation of a random series using generalized Silverman–Sass matrixes [1, 4], which are a special case of the Voronoi–Nörlund methods [1]. In view of the estimates for the transition functions of random series [2, 3], for each regular matrix $\widetilde{A_m}$ the switching conditions

$$\widetilde{A_m} \subseteq \widetilde{A_n}$$

and the necessary conditions for separability of the corresponding field of summability are found.

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GROWTH OF SOME RANDOM PROCESS

O. A. TYMOSHENKO, O. I. KLESOV

A lot of examples of stochastic differential equation applications can be found in engineering, physics, chemistry, biology, economics, financial mathematics etc. The behavior of solutions for all stochastic models is quite irregular which perhaps reflects the random nature of a solutions. However, one could want to have a simpler (possibly, non-random) approximation of the solution that explains the main trend of the fluctuations. I.I. Gihman and A.V. Skorokhod [1] are one of the who first began to study the problem of the asymptotic behavior of the solution of *autonomous* differential equation perturbed by Wiener process. In case, when solution of the stochastic differential equation is tend to infinity, they have found a deterministic function ϕ for which

$$\lim_{t\to\infty} \frac{\eta(t)}{\phi(t)} = 1 \quad \text{a.s.} \tag{1}$$

This function ϕ is called the exact order of growth of the process η as $t \to \infty$.

In the talk, the problem of finding nonrandom approximations (1) of solutions of a general class of stochastic differential equations that includes equations such that *one-factor short rate model of interest rates, affine model, constant elasticity of variance model, Gompertz equation* is studied. We follow the setting by I.I. Gihman and A.V. Skorohod [1], however the results of our talk are more general.

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ASYMPTOTICALLY EFFICIENT STATISTICAL ESTIMATION IN PARTIALLY OBSERVED SYSTEMS

V. ZAIATS

Assume that we observe a process $X = (X_t,\ 0 \le t \le T)$ satisfying the following system of stochastic differential equations:

$$\mathrm{d}X_t = h_t Y_t \,\mathrm{d}t + \varepsilon \,\mathrm{d}W_t, \quad X_0 = 0,$$
$$\mathrm{d}Y_t = g_t Y_t \,\mathrm{d}t + \varepsilon \,\mathrm{d}V_t, \quad Y_0 = y_0 \ne 0, \quad 0 \le t \le T,$$

where W_t and V_t, $0 \le t \le T$, are two independent Wiener processes. The process $Y = (Y_t,\ 0 \le t \le T)$ **is not observed** directly, but it is the one *that should be controlled*.

The problem of asymptotically efficient estimation of different functions on $0 \le t \le T$ under a *small noise*, i.e., as $\varepsilon \to 0$, is considered. An approach due to Kutoyants [1, Chapter 6] for handling this type of problems leads to constructing kernel-type estimators for the functions $f_t := h_t y_t$, h_t, y_t, g_t, $0 \le t \le T$. Here y_t, $0 \le t \le T$, stands for the solution of the above model with both noise terms dropped.

Lower bounds on the rate of convergence of asymptotically efficient estimators are obtained. The estimators giving these lower bounds are constructed explicitly.

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LIMIT THEOREMS FOR COMPOUND RENEWAL PROCESSES

N. M. ZINCHENKO

We consider *compound renewal processes* (random sums, randomly stopped sums) of the form

$$D(t) = S(N(t)) = \sum_{i=1}^{N(t)} X_i,$$

where $\{X_i, i \geq 1\}$ are i.i.d.r.v., $S(t) = \sum_{i=1}^{[t]} X_i$, $t > 0$, $S(0) = 0$; $\{Z_i, i \geq 1\}$ is another sequence of non-negative i.i.d.r.v. independent of $\{X_i\}$, $Z(x) = \sum_{i=1}^{[x]} Z_i$, $x > 0$, $Z(0) = 0$ and *renewal (counting) process* $N(t)$ is defined as $N(t) = \inf\{x \geq 0 : Z(x) > t\}$.

A few classes of strong limit theorems for compound renewal processes are investigated. The first one is so-called *strong invariance principle* (SIP), which presents the sufficient conditions for a.s. approximation of $D(t)$ by a Wiener or α-stable Levy process under various assumptions on the renewal process $N(t)$ and *independent* summands $\{X_i, i \geq 1\}$. For instance, the following theorem is proved:

Theorem 1. *(i) Let $E|X_1|^{p_1} < \infty$, $E|Z_1|^{p_2} < \infty$ and suppose that $p = \min\{p_1, p_2\} > 2$, $EX_1 = m$, $VarX_1 = \sigma^2$, $EZ_1 = 1/\lambda > 0$, $\tau^2 = varZ_1$, then $\{X_i\}$ and $N(t)$ can be constructed on the same probability space together with a standard Wiener process $\{W(t), t \geq 0\}$ in such a way that a.s.*

$$\sup_{0 \leq t \leq T} |D(t) - \lambda m t - \nu W(t)| = o(T^{1/p}), \quad \nu^2 = \sigma^2 \lambda + m^2 \tau^2 \lambda^3; \quad (1)$$

(ii) if $p = 2$, then right side of (1) is $o(T \ln \ln T)^{1/2}$;
(iii) if $E\exp(uX_1) < \infty$, $E\exp(uZ_1) < \infty$ for all $u \in (0, u_o)$, then right-hand side of (1) is $O(\ln T)$.

Corresponding proofs are based on the rather general SIP-type theorems for the superposition of random processes (not obligatory connected with partial sums) [1]. The same approach is used for exploration the asymptotic behavior of $D(t)$ in the cases, when summands are *dependent* (φ-mixing, weakly dependent, associated).

On the next step we use SIP-type results for further investigation the rate of grows of compound renewal processes and their increments. For this purpose a number of integral tests are proposed. As a consequence various modifications of the LIL and Erdös-Rényi-Csörgő-Révész-type law for increments $D(t + a_t) - D(t)$ over intervals, whose length a_t grows as $t \to \infty$, are obtained under various dependent and moment assumptions on $\{X_i, i \geq 1\}$ and $\{Z_i, i \geq 1\}$.

Certain applications in risk and queuing theories are discussed [2].

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