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OPERATION CALCULATION

Didactic material for a modal reference work on mathematical analysis for students of 2nd year engineering faculties

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Kiev 2016

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Operational calculus.

Control work.

Introduction

Operational calculus is studied in the course of mathematical analysis in the third semester. Surgery (symbolic) calculus widely used in various fields of science and technology. A particularly important role it plays in the study of transients in linear physical systems theory of electrical circuits, automation, radio engineering, mechanics.

Didactic material contains 30 variants of modular control work being done by the second year students of technical specialties in the third semester. The work consists of five tasks and is designed for 90 minutes.

In the first task, you find the present original image. It helps to learn the definition of the Laplace transform and its properties. The second task of the present must Find image of the original. In the third task, you solve the Cauchy problem for linear differential equations with piecewise continuous right-hand side. The fourth task is proposed to solve the Cauchy problem for linear differential equations using Duhamel integral. In the fifth task proposed Volterra integral equation of convolution type. Each version of the control module attached reply.

The use of of operational calculations

I. Solution of Cauchy problem for linear differential equation with constant coefficients in finding the right part of image

According to the plan:

1. By means of Laplace transformations upgrade linear differential equation in relative algebraic image
2. Find out in this algebraic equation of the desired image of the original (called operational solution)
3. According to reproduce the original image (answer)

Example 1

$y''+2y'+y = \sin t \quad y(0)=0,\ y'(0) = -1$

$y(t)\to Y(p)$

$y'(t)\to pY(p) - y(0) = pY(p) - 0 = pY(p)$

$y''(t)\to p^2Y(p) - py(0) - y'(0) = p^2Y(p)+1$

$\sin t\to \frac{1}{p^2+1}$

have the operator equation

$p^2Y(p)+1+2pY(p)+Y(p) = \frac{1}{p^2+1}$

$Y(p)(p^2+2p+1) = -1+\frac{1}{p^2+1}$

$Y(p) = -\frac{1}{(p+1)^2}+\frac{1}{(p+1)^2(p^2+1)}$ (operational solution)

Find the original:

a) $-\frac{1}{(p+1)^2} = \left(\frac{1}{p+1}\right)' \to -te^{-t}$

applied the theorem of differentiation of the original

$F'(p)\to -tf(t)$

We have $F(p)=\frac{1}{p+1}\to e^{-t}$

b) relative to the second term we apply the second theorem of decomposition

$$\frac{1}{(p+1)^2(p^2+1)} = \frac{1}{(p+1)^2(p+i)(p-i)}$$

P_1= -i Pole of II order

P_2= -i simple pole

P_3= i simple pole

$$F(p) = \frac{1}{(p+1)^2(p^2+1)} = \underset{p=p_1}{res} F(p)e^{pt} + \underset{p=p_2}{res} F(p)e^{pt} + \underset{p=p_3}{res} F(p)e^{pt} =$$

$$=\lim_{p\to -1}\frac{1}{(2-1)!}\cdot\frac{d}{dp}\left(\frac{e^{pt}}{p^2+1}\right)+\lim_{p\to i}\frac{e^{pt}}{(p+1)^2(p+i)}+\lim_{p\to -i}\frac{e^{pt}}{(p+1)^2(p-i)} =$$

$=\lim_{p\to -1} \frac{te^{pt}(p^2+1)-e^{pt}\cdot 2p}{(p^2+1)^2}+\frac{e^{it}}{2i(i+1)^2}+\frac{e^{-it}}{-2i(-i+1)^2}=\frac{te^{-t}2+2e^{-t}}{4}+\frac{e^{it}}{2i\cdot 2i}+\frac{e^{-it}}{-2i(-2i)}=$

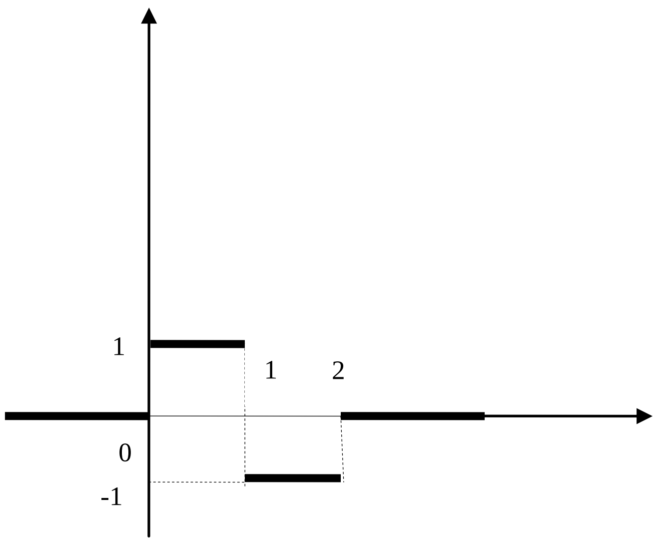
$=\frac{1}{2}te^{-t}+\frac{1}{2}e^{-t}-\frac{1}{2}\frac{e^{it}+e^{-it}}{2}=\frac{1}{2}te^{-t}+\frac{1}{2}e^{-t}-\frac{1}{2}\cos t$

as the solution of a linear differential equation is a function of:

$y(t) = -te^{-t}+\frac{1}{2}te^{-t}+\frac{1}{2}e^{-t}-\frac{1}{2}\cos t = \frac{1}{2}(e^{-t}-te^{-t}-\cos t)$

Example 2

$y''(t) + y(t)= f(t)$, where $f(t)$ given graphically, $y(0)=y'(0)=0$



The solution

$y(t)\to Y(p)$

$y''(t) \to p^2Y(p)-py(o)-y'(0)=p^2Y(p)$

$f(t)=\eta(t) - \eta(t-1)-\eta(t-1)+\eta(t-2)=$

$=\eta(t)-2\eta(t-1)+\eta(t-2)\to \frac{1}{p}-\frac{2}{p}e^{-p}+\frac{1}{p}e^{-2p}$

have the operational equation

$P^2Y(p)+Y(p)=\frac{1}{p}-\frac{2}{p}e^{-p}+\frac{1}{p}e^{-2p}$

$Y(p)=\frac{1}{p(p^2+1)}-\frac{2e^{-p}}{p(p^2+1)}+\frac{1}{p(p^2+1)}e^{-2p}$

This is the solution in the operational form, find the original:

a) $F(p) = \frac{1}{p(p^2+1)}=\frac{A}{P}+\frac{Mp+N}{p^2+1} = \frac{1}{p}-\frac{p}{p^2+1}\to$

$\to\eta(t)-\cos t\cdot\eta(t) = (1-\cos t)\eta(t)=2\sin^2\frac{t}{2}\eta(t)$

b) Availability multipliers e^{-pa} points to the possibility of applying theorem of delay:

$e^{-pa}F(p) \to f(t-a)\cdot\eta(t-a)$

because

$$\frac{2}{p(p^2+1)}e^{-p} \to 4\sin^2\frac{t-1}{2}\cdot\eta(t-1)$$

$$\frac{1}{p(p^2+1)}e^{-2p} \to 2\sin^2\frac{t-2}{2}\cdot\eta(t-2)$$

Answer:

$$y(t) = 2\sin^2\frac{t}{2}\cdot\eta(t) - 4\sin^2\frac{t-1}{2}\cdot\eta(t-1) + 2\sin^2\frac{t-2}{2}\cdot\eta(t-2)$$

Note: function y (t) will satisfy the equation at all points where it is continuous.

II. The solution of the Cauchy problem without finding image of the right side

Example 3

$$y''(t) = \frac{1}{1+t^2} \qquad y(0)=y'(0)=0$$

The solution

If $\frac{1}{1+t^2} \to F(p)$ (where F(p) – (some unknown images)

Then the operational equation is:

$p^2Y(p)=F(p)$

$Y(p)=\frac{1}{p^2}F(p)$ – operational solution

Operator solution got as a product of two images, by Borel theorem we have:

$G(p)\cdot F(p) \to g(t)*f(t)$

We have $G(p)=\frac{1}{p^2} \to t$, $\qquad F(p) \to \frac{1}{1+t^2}$, so

$$Y(p)=\frac{1}{p^2}\cdot F(p)=G(p)\cdot F(p) \to g(t)*f(t)=$$

$$=\int_0^t g(t-\tau)f(\tau)d\tau = t*\frac{1}{1+t^2} = \int_0^t (t-\tau)\cdot\frac{1}{1+\tau^2}d\tau = t\int_o^t \frac{d\tau}{1+\tau^2} -$$

$$-\int_0^t \frac{\tau d\tau}{1+\tau^2} = t\,arctg\,\tau \Big/_0^t - \frac{1}{2}\ln(1+\tau^2)\Big/_0^t = t\,arctg\,t - \frac{1}{2}\ln(1+t^2) = y(t)$$

Answer: $y(t)=t\,arctg\,t - \frac{1}{2}\ln(1+t^2)$

Note:

1. The requirement of setting the initial conditions at the point t = 0 is not essential, as the linear change of variables y = (-a new variable) Cauchy problem at $t = t0 \neq 0$ is reduced to the Cauchy problem with initial conditions at the point.
2. Similarly, the replacement of unknown function problem with nonzero initial conditions can be reduced to a problem with zero initial conditions.

For example, if the initial conditions $y(0)=y_0 \quad y'(0)=y_1$

then the replacement of the function y (t) to z (t), where z (t)

$= y(t) - y_0 - y_1 t$

we obtain: $z(0) = 0$

$z'(0) = y'(t) - y_1 /_{t=0} = 0$

3. If the initial conditions y0, y1, y2, yn-1 is not considered a given, but arbitrary constants, then y (t) is not a solution of the Cauchy problem it is the general solution of the differential equation.

III. Solving systems of linear differential equations with constant coefficients.

Systems of linear differential equations are solved similarly, the difference is that we obtain a system of operational equations.

Example 4

$$\begin{cases} x'=x+3y \\ y'=x-y \end{cases} \qquad \begin{matrix} x=x(t) \\ y=y(t) \end{matrix} \qquad \begin{matrix} \text{Initial conditions} \\ x(0)=1, \quad y(0)=0 \end{matrix}$$

The solution

$x(t) \to X(p)$ $\qquad$ $x'(t) \to pX(p)-x(0)=pX(p)-1$

$y(t) \to Y(p)$ $\qquad$ $y'(t) \to pY(p)-y(0)=pY(p)$

The system of operational equations:

$pX(p)-1=X(p)+3Y(p)$

$pY(p)=X(p)-Y(p)$

rewrite the system:

$$\begin{cases} (p-1)X-3Y=1 & X=\dfrac{\Delta x}{\Delta} \\ X-(p+1)Y=0 & Y=\dfrac{\Delta y}{\Delta} \end{cases}$$

$$\Delta = \begin{vmatrix} p-1 & -3 \\ 1 & -(p+1) \end{vmatrix} = -(p^2-1)+3 = -(p^2-4)$$

$$\Delta x = \begin{vmatrix} 1 & -3 \\ 0 & -(p+1) \end{vmatrix} = -(p+1)$$

$$\Delta y = \begin{vmatrix} p-1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$X=\frac{\Delta x}{\Delta}=\frac{p+1}{p^2-4}=\frac{p}{p^2-4}+\frac{1}{p^2-4} \to ch2t+\frac{1}{2}sh2t$$

$$Y=\frac{\Delta y}{\Delta}=\frac{1}{p^2-4}=\frac{1}{2}\cdot\frac{2}{p^2-4} \to \frac{1}{2}sh2t$$

Answer:

$$\begin{cases} x(t)=ch2t+\dfrac{1}{2}sh2t \\ y(t)=\dfrac{1}{2}sh2t \end{cases}$$

IV. The solution of integral equations Voltaire of the I and II order

Integral equation is called the equation that contains the required function under the integral sign.

Consider a simple integral equations of Voltaire wich type is convolution

I kind: $\int_0^t k(t-\tau)y(\tau)d\tau = f(t)$

II kind: $y(t)+\int_0^t k(t-\tau)y(\tau)d\tau = f(t)$

where y (t) - desired function
f (t) - known function
k (t-) - known function, called the nucleus, and depend on the difference of arguments.
If the function k (t-), f (t) are functions - originals, then using operational calculations can find the solution of the integral equation.
let $y(t)\to Y(p)$
$f(t)\to F(p)$
$k(t-\tau)\to K(p)$
then in operational form of the first equation is$K(p)\cdot Y(p) = F(p)$

$$Y(p) = \frac{F(p)}{K(p)} \to y(t)$$

The second equation:$Y(p)+K(p)\cdot Y(p) = F(p)$

$$Y(p)=\frac{F(p)}{1+K(p)} \to y(t)$$

In both cases used the theorem of Borel about image convolution of two functions.

Example 5

$y(x) = \sin x+\int_0^x (x-t)y(t)dt$ integral equation of the II kind

$$Y(p) = \frac{1}{p^2+1}+\frac{1}{p^2}\cdot Y(p)$$

$$Y(p) = \frac{p^2}{(p^2+1)(p^2-1)} = \frac{1}{2}\left(\frac{1}{p^2-1}+\frac{1}{p^2+1}\right) \to \frac{1}{2}(shx+\sin x), x>0$$

Answer: $y(x)= \frac{1}{2}(shx+\sin x), x>0$

Example 6

$\int_0^t \cos(t-\tau)y(\tau)d\tau = \sin t$ integral equation of the I kind

$\cos t * y(t) = \sin t$

$$\frac{p}{p^2+1}\cdot Y(p) = \frac{1}{p^2+1}$$

$$Y(p)=\frac{1}{p} \to 1$$

Answer: $y(t)=1, \quad t>0$

The structure of short-term module control work SCW – 3

1. Find the Laplace transform of the function - the original
2. Find image of the original data by Laplace

   Solve linear differential equation with operational method

   (45 minutes)

Structure of Module control work MCW – 3

1. Find the Laplace transform of the function - the original
2. Find image of the original data by Laplace
3. Solve the Cauchy problem for linear differential equations by operational method
4. Solve the Cauchy problem for a linear differential equation using Duhamel formula
5. To solve the integral equation of convolution operational method

(90 minutes)

For example:

1. $f(t)=\sin^2 3t\cdot e^{2t}=\frac{1-\cos 6t}{2}\cdot e^{2t}\to\frac{1}{2}\left(\frac{1}{p-2}-\frac{p-2}{(p-2)^2+36}\right)$

2. $F(p) = \frac{p}{p^2-2p+5}=\frac{p-1+1}{(p-1)^2+4}=\frac{p-1}{(p-1)^2+4}+\frac{1}{(p-1)^2+4}\to e^t\cos 2t+\frac{1}{2}e^t\sin 2t$

3. $y''+9y=\eta(t-5)$

$y(0) = y'(0) = 0$

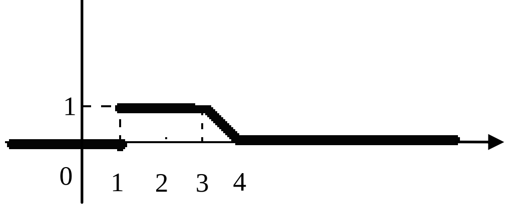
$p^2Y(p) + 9Y(p)=\frac{1}{p}e^{-5p}$

$Y(p) = \frac{1}{p(p^2+9)}\cdot e^{-5p}=\frac{1}{9}e^{-5p}\left(\frac{1}{p}-\frac{p}{p^2+9}\right)$

так як $\frac{1}{p}-\frac{p}{p^2+9}\to 1-\cos 3t=2\sin^2\frac{3t}{2}$

$y(t) = \frac{2}{9}\sin^2\frac{3(t-5)}{2}\eta(t-5)$

4. Find an image graphically of a given function f (t)



$f(t) = \eta(t-1) - \eta(t-3) + (-t+4)\eta(t-3) – (-t+4)\eta(t-4)=$

$= \eta(t-1)-(t-3)\eta(t-3)-(t-4)\eta(t-4) \to$

$\to\frac{1}{p}e^{-p}-\frac{1}{p^2}e^{-3p}-\frac{1}{p^2}e^{-4p}$

Example and solution of the problem «operator calculus».

1. Find the image function.
   $f(t)=e^{4(t-5)}\cos(t-5)\eta(t-5)$
2. Find original of the following image
   $F(p)=\dfrac{3p}{p^2-4}$
3. Solve the Cauchy problem by operational calculations
   $y''+2y'+y=f(t),\quad y(0)=y'(0)=0,$
   $f(t)=\begin{cases}1, & 0\le t\le 1\\ 0, & t>1\end{cases}$
4. Using the formula Duhamel solve the solution of equation.
   $y''-2y'+y=\dfrac{e^t}{1+t^2},\quad y(0)=y'(0)=0$
5. Solve the integral equation
   $y(t)+\int\limits_0^t e^{t-\tau}y(\tau)d\tau=\cos 2t$

Exercise 1. Solving.

As you know $\cos t\to\dfrac{p}{p^2+1}$, namely $\cos t\cdot\eta(t)\to\dfrac{p}{p^2+1}$. Because shift theorem:

$e^{4t}\cos t\cdot\eta(t)\to\dfrac{p-4}{(p-4)^2+1}$, because lag theorem

$e^{4(t-5)}\cos(t-5)\cdot\eta(t-5)\to\dfrac{p-4}{(p-4)^2+1}\cdot e^{-5p}$.

Answer: $F(p)=\dfrac{p-4}{(p-4)^2+1}\cdot e^{-5p}$.

Exercise 2. Solving.

We know that $\dfrac{p}{p^2-4}\to ch2t$, by virtue of linear property, $\dfrac{3p}{p^2-4}\to 3ch2t$.

Answer: $F(p)=\dfrac{3p}{p^2-4}$.

Exercise 3. Solving.

The right side of equation $f(t)$ is piecewise continuous function. We write its analytical expression:

$f(t)=\eta(t)-\eta(t-1)$. By using the linearity property and applying lag theorem $f(t)\to\frac{1}{p}-\frac{1}{p}e^{-p}$.

Let $y(t)\to Y(p)$. Then $y'(t)\to pY(p)$, $y''(t)\to p^2Y(p)$.

Let us write the operator equation

$p^2Y(p)+2pY(p)+Y(p)=\frac{1}{p}-\frac{1}{p}e^{-p}$, where $Y(p)=\frac{1}{p(p+1)^2}-\frac{1}{p(p+1)^2}e^{-p}$. We found the original of the image. For table image have: $\frac{1}{p+1}\to e^{-t}$.

By the image differencing theoreme we have $\left(\frac{1}{p+1}\right)'\to -te^{-t}\Rightarrow -\frac{1}{(p+1)^2}\to -te^{-t}\Rightarrow\frac{1}{(p+1)^2}\to te^{-t}$. By the integration of original theorem:

$\frac{1}{p(p+1)^2}\to\int_0^t\tau e^{-\tau}d\tau=1-e^{-t}-te^{-t}$. Therefore $\frac{1}{p(p+1)^2}\to\left(1-e^{-t}-te^{-t}\right)\eta(t)$. Account for lag theorem we obtain $\frac{1}{p(p+1)^2}e^{-p}\to\left(1-e^{-(t-1)}-(t-1)e^{-(t-1)}\right)\eta(t-1)$.

Answer: $y(t)=\left(1-e^{-t}-te^{-t}\right)\eta(t)-\left(1-e^{-(t-1)}-(t-1)e^{-(t-1)}\right)\eta(t-1)$.

Exercise 4. Solving.

Find a solution $y_1(t)$ subsidiary equation $y''-2y'+y=1$ by initial condition $y(0)=y'(0)=0$.

Let $y_1(t)\to Y_1(p)$. Then $y_1'(t)\to pY_1(p)$, $y_1''(t)\to p^2Y_1(p)$. Since $1\to\frac{1}{p}$, we get operator equation: $p^2Y_1(p)-2pY_1(p)+Y_1(p)=\frac{1}{p}$, from here $Y_1(p)=\frac{1}{p(p-1)^2}$.

With the resulting image we find the original. This can be done in different ways. First way. By expansion theorem

$$y_1(t)=\operatorname*{res}_{p=0}y_1(t)e^{pt}+\operatorname*{res}_{p=1}y_1(t)e^{pt}=\lim_{p\to 0}\frac{e^{pt}p}{p(p-1)^2}+\lim_{p\to 1}\left(\frac{e^{pt}(p-1)^2}{p(p-1)^2}\right)'=$$

$$=1+\lim_{p\to 1}\frac{te^{pt}p-e^{pt}}{p^2}\ y_1'(t)=-e^t+e^t+te^t=te^t$$

$$te^t\to\frac{1}{p(p-1)^2}\to 1+te^t-e^t$$

$$\frac{1}{p(p-1)^2}=\frac{1}{p}-\frac{1}{p-1}+\frac{1}{(p-1)^2}$$

$$y(t)=\int_0^t\frac{e^\tau}{1+\tau^2}(t-\tau)e^{t-\tau}d\tau=e^t\int_0^t\frac{t-\tau}{1+\tau^2}d\tau=e^t\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^2\right)\right).$$

$$y(t)=\int_0^t\frac{e^\tau}{1+\tau^2}(t-\tau)e^{t-\tau}d\tau=e^t\int_0^t=e^t\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^2\right)\right)$$

$$y(t)=e^t\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^2\right)\right)$$

$$\int_0^t e^{t-\tau}y(\tau)d\tau=e^t*y(t)$$

Second way.

Table image should: $te^t\to\frac{1}{(p-1)^2}$. By the theorem of original integration we get

$$\frac{1}{p(p-1)^2}\to\int_0^t\tau e^\tau d\tau=1+te^t-e^t.$$

Third way.

Decompose proper rational fraction into a sum of simple fractions:

$$\frac{1}{p(p-1)^2}=\frac{1}{p}-\frac{1}{p-1}+\frac{1}{(p-1)^2}.$$

From table images we obtain $\frac{1}{p(p-1)^2}\to 1+te^t-e^t$. Since $y_1(t)=1+te^t-e^t$.

We have $y_1'(t)=-e^t+e^t+te^t=te^t$. By Duhamel formula: $y(t)=\int_0^t f(\tau)y_1'(\tau)d\tau$.

Since, $y(t)=\int_0^t\frac{e^\tau}{1+\tau^2}(t-\tau)e^{t-\tau}d\tau=e^t\int_0^t\frac{t-\tau}{1+\tau^2}d\tau=e^t\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^2\right)\right)$.

Ansver: $y(t)=e^t\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^2\right)\right)$.

Exercise 5. Solving.

Considering that $\int_0^t e^{t-\tau}y(\tau)d\tau$ is convolution function e^t and $y(t)$.

Let $y(t)\to Y(p)$. Table image should: $e^t\to\frac{1}{p-1}$, $\cos 2t\to\frac{p}{p^2+4}$, by Borel theorem $e^t*y(t)\to\frac{Y(p)}{p-1}$. So that, operator equation is written as:

$Y(p)+\frac{Y(p)}{p-1}=\frac{p}{p^2+4}$, and its solution $Y(p)=\frac{p-1}{p^2+4}$ can be represented as $Y(p)=\frac{p}{p^2+4}-\frac{1}{p^2+4}=\frac{p}{p^2+4}-\frac{1}{2}\frac{2}{p^2+4}$. From the table we Find image of the original $y(t)=\cos 2t-\frac{1}{2}\sin 2t$.

Answer: $y(t)=\cos 2t-\frac{1}{2}\sin 2t$.

Variant 1

1. Find image of the original $f(t)=\dfrac{e^{-2t}\sin^2 t}{t}$.

2. Find original of the following image $F(p)=\dfrac{2p+1}{(p-2)^2}$.

3. Solve the Cauchy problem by operational calculations $y''+4y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases} t, & 0\le t<1 \\ 2-t, & 1\le t<2 \\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y=tht,\ y(0)=y'(0)=0$.

5. Solve the integral equation $\int_0^t y(\tau)\sin(t-\tau)\,d\tau=1-\cos t$.

Variant 2

1. Find image of the original $f(t)=e^{4(t-5)}\cos(t-5)\eta(t-5)$.

2. Find original of the following image $F(p)=\dfrac{3p}{p^2-4}$.

3. Solve the Cauchy problem by operational calculations $y''+2y'+y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases} 1, & 0\le t\le 1 \\ 0, & t>1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-2y'+y=\dfrac{e^t}{1+t^2},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)+\int_0^t e^{t-\tau}y(\tau)\,d\tau=\cos 2t$.

Variant 3

1. Find image of the original $f(t)=t\sin 5t$.
2. Find original of the following image $F(p)=\dfrac{p^3+p^2-2p+1}{p^5-2p^4+p^3}$.
3. Solve the Cauchy problem by operational calculations $y''+2y'+2y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t\le 2\\ 0, & t>2\end{cases}$.
4. Using the integral Duhamel solve the Cauchy problem $y''-y'=\dfrac{1}{1+e^t},\ y(0)=y'(0)=0$.
5. Solve the integral equation $\int_0^t y(\tau)(t-\tau)^2\,d\tau=\dfrac{t^3}{3}$

Variant 4

1. Find image of the original $f(t)=e^{-4t}\cos 2t\cos 5t$.
2. Find original of the following image $F(p)=\dfrac{1}{(p^2+4)(p^2+9)}$.
3. Solve the Cauchy problem by operational calculations $y'+4y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}3, & 0\le t<3\\ 6, & t>3\end{cases}$.
4. Using the integral Duhamel solve the Cauchy problem $y''-2y'+2y=2e^t\cos t,\ y(0)=y'(0)=0$.

5. Solve the integral equation $-\int_0^t y(\tau)(t-\tau)\, d\tau + y(t) = \sin t$.

Variant 5

1. Find image of the original $f(t) = \dfrac{\cos 6t - \cos 2t}{t}$.

2. Find original of the following image $F(p) = \dfrac{pe^{-2p}}{p^2+16}$.

3. Solve the Cauchy problem by operational calculations $y' + y = f(t),\ y(0) = 0$ де $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y'' - y = th^2 t,\ y(0) = y'(0) = 0$.

5. Solve the integral equation $\int_0^t y(\tau)\sin(t-\tau)\, d\tau = \sin^2 t$.

Variant 6

1. Find image of the original $f(t) = \dfrac{\sin^2 t}{t}$.

2. Find original of the following image $F(p) = \dfrac{1}{(p^2-4)(p-1)}$.

3. Solve the Cauchy problem by operational calculations $y' + 3y = f(t),\ y(0) = 1$ де $f(t) = \begin{cases} 2, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y'' - y = \dfrac{1}{cht},\ y(0) = y'(0) = 0$.

5. Solve the integral equation $\int_0^t y(\tau)\cos(t-\tau)\ d\tau = t+t^2$.

Variant 7

1. Find image of the original $f(t)=\dfrac{\sin 7t\cdot\sin 3t}{t}$.

2. Find original of the following image $F(p)=\dfrac{1}{(p-1)^2(p+1)}$.

3. Solve the Cauchy problem by operational calculations $y'+2y=f(t),\ y(0)=3$ де $f(t)=\begin{cases}2, & 0\le t<1\\ 1, & t\ge 1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y'=\dfrac{e^t}{1+e^t},\ \ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t+\dfrac{1}{2}\int_0^t (t-\tau)^2\, y(\tau)d\tau$.

Variant 8

1. Find image of the original $f(t)=\int_0^t sh\tau d\tau$.

2. Find original of the following image $F(p)=\dfrac{1}{p^3-8}$.

3. Solve the Cauchy problem by operational calculations

$$y''+y=f(t),\ y(0)=y'(0)=0 \text{ де } f(t)=\begin{cases}1, & 0\le t<1\\ -1, & 1\le t<2\\ 0, & t\ge 2\end{cases}.$$

4. Using the integral Duhamel solve the Cauchy problem $y'' - 2y' + y = \frac{e^t}{1+t}$, $y(0) = y'(0) = 0$.

5. Solve the integral equation $y(t) - \int_0^t y(\tau)\sin(t-\tau)\, d\tau = t$.

Variant 9

1. Find image of the original $f(t) = sh2t\cos 3t$.

2. Find original of the following image $F(p) = \frac{p+3}{p\left(p^2 - 4p + 3\right)}$.

3. Solve the Cauchy problem by operational calculations $y' + y = f(t),\ y(0) = 0,$ де $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y' - y = \frac{1}{3+e^t}$, $y(0) = 0$.

5. Solve the integral equation $y(t) = \cos t + \int_0^t e^{t-\tau} y(\tau)\, d\tau$.

Variant 10

1. Find image of the original $f(t) = \cos^3 t$.

2. Find original of the following image $F(p) = \frac{1}{p\left(p^2 + 4\right)\left(p^2 + 1\right)}$.

3. Solve the Cauchy problem by operational calculations $y''+y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t<1\\ -1, & 1\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y=\frac{1}{1+cht},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=1+t+\int_0^t \cos(t-\tau)y(\tau)\,d\tau$.

Variant 11

1. Find image of the original $f(t)=\frac{1-e^{-4t}}{te^t}$.

2. Find original of the following image $F(p)=\frac{p}{(p^2+2)^2}$.

3. Solve the Cauchy problem by operational calculations $y''+y'=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}2, & 0\le t<1\\ 1, & 1\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+y=\frac{1}{1+e^t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=\frac{t^2}{2}+\int_0^t (t-\tau)e^{-(t-\tau)}d\tau$.

Variant 12

1. Find image of the original $f(t)=\int_0^t \frac{\sin\tau}{\tau}d\tau$.

2. Find original of the following image $F(p)=\frac{e^{-3p}}{p(p-1)}$.

3. Solve the Cauchy problem by operational calculations $y''+y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1-t, & 0\le t<1\\ 0, & t\ge 1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-4y=\frac{1}{ch^3 2t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=e^{-t}+\frac{1}{2}\int_0^t (t-\tau)^2 y(\tau)d\tau$.

Variant 13

1. Find image of the original $f(t)=sh2t\cdot\sin 5t$.

2. Find original of the following image $F(p)=\frac{1}{p(p^2+1)(p^2+4)}$.

3. Solve the Cauchy problem by operational calculations $y''+4y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}t, & 0\le t<1\\ 0, & t\ge 1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y=\frac{1}{ch^2 t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t+2\int_0^t ((t-\tau)-\sin(t-\tau))y(\tau)d\tau$.

Variant 14

1. Find image of the original $f(t)=e^{-t}\sin 2t\cdot\cos t$.

2. Find original of the following image $F(p)=\dfrac{3p+1}{(p-5)^6}$.

3. Solve the Cauchy problem by operational calculations $y''-y'=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t<1\\ 2, & 1\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+y'=\dfrac{e^t}{1+e^t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=\sin t+2\int_0^t\cos(t-\tau)y(\tau)d\tau$.

Variant 15

1. Find image of the original $f(t)=e^{-3t}t^8$.

2. Find original of the following image $F(p)=\dfrac{5p+1}{\left(p^2+9\right)^2}$.

3. Solve the Cauchy problem by operational calculations $y''-4y'=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t<1\\ -1, & 1\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+2y'+y=\dfrac{e^{-t}}{(1+t)^2},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=1+\dfrac{1}{2}\int_0^t\sin(t-\tau)y(\tau)d\tau$.

Variant 16

1. Find image of the original $f(t)=e^{4(t-5)}\cos(t-5)\eta(t-5)$.

2. Find original of the following image $F(p)=\dfrac{p+2}{p^2-5p+6}$.

3. Solve the Cauchy problem by operational calculations

$$y''+y=f(t),\ y(0)=y'(0)=0,\ \text{де } f(t)=\begin{cases}1, & 0\le t<1\\ -1, & 1\le t<2\\ 0, & t\ge 2\end{cases}.$$

4. Using the integral Duhamel solve the Cauchy problem

$$y''-y=\frac{1}{ch^3 t},\ \ y(0)=y'(0)=0 .$$

5. Solve the integral equation $y(t)=e^{-t}-2\int_0^t \cos(t-\tau)y(\tau)d\tau$.

Variant 17

1. Find image of the original $f(t)=ch2t\cdot\sin 3t$.

2. Find original of the following image $F(p)=\dfrac{1}{(p^2+1)(p^2+9)}$.

3. Solve the Cauchy problem by operational calculations

$$y''+3y=f(t),\ y(0)=y'(0)=0,\ \text{де } f(t)=\begin{cases}1, & 0\le t<4\\ 0, & t\ge 4\end{cases}.$$

4. Using the integral Duhamel solve the Cauchy problem

$$y''-y'=\frac{e^{2t}}{(1+e^t)^2},\ \ y(0)=y'(0)=0 .$$

5. Solve the integral equation $y(t)=1+\frac{1}{6}\int_0^t (t-\tau)^3 y(\tau)d\tau$.

Variant 18

1. Find image of the original $f(t)=t\cdot\sin 4t$.

2. Find original of the following image $F(p)=\dfrac{e^{-2p}}{(p-1)^2}$.

3. Solve the Cauchy problem by operational calculations $y''+y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t<2\\ 4, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+2y'+y=\dfrac{te^{-t}}{t+1},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t-\int_0^t sh(t-\tau)y(\tau)d\tau$.

Variant 19

1. Find image of the original $f(t)=\sin t\cdot\eta(t-\pi)$.

2. Find original of the following image $F(p)=\dfrac{1}{p^4-5p^2+4}$.

3. Solve the Cauchy problem by operational calculations $y''+4y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}1, & 0\le t<\pi\\ 0, & t\ge \pi\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y'=\dfrac{e^{2t}}{e^t+2},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=sht-\int_0^t ch(t-\tau)y(\tau)d\tau$.

Variant 20

1. Find image of the original $f(t)=sht\cdot\cos 3t$.

2. Find original of the following image $F(p)=\dfrac{p+3}{p(p^2-4p+3)}$.

3. Solve the Cauchy problem by operational calculations $y''+9y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}0, & 0\le t<3\\ 1, & t\ge 3\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y'=\dfrac{sht}{ch^2 t},\ \ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=\sin t+\int_0^t (t-\tau)y(\tau)d\tau$.

Variant 21

1. Find image of the original $f(t)=\int_0^t \sin^2\tau d\tau$.

2. Find original of the following image $F(p)=\dfrac{1}{p^3(p-1)}$.

3. Solve the Cauchy problem by operational calculations $y''+4y=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}0, & t<2\\ 2, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+y'=\dfrac{e^t}{(e^t+1)^2},\ \ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t-\int_0^t e^{t-\tau}y(\tau)d\tau$.

Variant 22

1. Find image of the original $f(t)=(2t+1)\cos 3t$.

2. Find original of the following image $F(p)=\dfrac{1}{p(p^2+1)}$.

3. Solve the Cauchy problem by operational calculations $y'+2y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}2, & 0\le t<1\\ 1, & t\ge 1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+2y'+y=\dfrac{e^{-t}}{t^2+1},\ y(0)=y'(0)=0$.

5. Solve the integral equation $t=\int_0^t ch(t-\tau)y(\tau)d\tau$.

Variant 23

1. Find image of the original $f(t)=t\cdot e^{-t}cht$.

2. Find original of the following image $F(p)=\dfrac{1}{(p^2+1)^2}$.

3. Solve the Cauchy problem by operational calculations $y''+3y'=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}0, & t<2\\ 1, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-2y'+y=\dfrac{e^{t}}{ch^2t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t-\int_0^t sh(t-\tau)y(\tau)d\tau$.

Variant 24

1. Find image of the original $f(t)=(t+1)\sin 2t$.

2. Find original of the following image $F(p)=\dfrac{e^{-3p}}{(p+1)^3}$.

3. Solve the Cauchy problem by operational calculations $y''+3y'=f(t),\ y(0)=y'(0)=0,$ де $f(t)=\begin{cases}0, & t<1\\ 1, & t\ge 1\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+2y'+y=\dfrac{e^{-t}}{ch^2 t},\ y(0)=y'(0)=0$.

5. Solve the integral equation $t+t^2=\int_0^t \cos(t-\tau)y(\tau)d\tau$.

Variant 25

1. Find image of the original $f(t)=t^2e^{3t}$.

2. Find original of the following image $F(p)=\dfrac{1}{p^2-p+7}$.

3. Solve the Cauchy problem by operational calculations $y'+3y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}1, & 0\le t<1\\ -1, & 1\le t<2\\ 1, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+2y'+y=\dfrac{e^{-t}}{t+1},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=sht+\int_0^t (t-\tau)y(\tau)d\tau$.

Variant 26

1. Find image of the original $f(t) = t \cdot \cos^2 t$.

2. Find original of the following image $F(p) = \dfrac{3e^{-4p}}{p^2+9}$.

3. Solve the Cauchy problem by operational calculations $y'' + 4y = f(t),\ y(0) = y'(0) = 0,$ де $f(t) = \begin{cases} 0, & t < 2 \\ 2, & t \ge 2 \end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y'' - y = \dfrac{1}{e^t + 3},\ y(0) = y'(0) = 0$.

5. Solve the integral equation $y(t) = 1 + 2\int_0^t \cos(t-\tau)\, y(\tau)\, d\tau$.

Variant 27

1. Find image of the original $f(t) = \int_0^t (t-\tau)^2 \sin\tau\, d\tau$.

2. Find original of the following image $F(p) = \dfrac{e^{-2p}}{p^2}$.

3. Solve the Cauchy problem by operational calculations $y' + 2y = f(t),\ y(0) = 3$ де $f(t) = \begin{cases} 2, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y'' + y = \dfrac{1}{e^t + 1},\ y(0) = y'(0) = 0$.

5. Solve the integral equation $y(t) = 1 + t + \int_0^t \sin(t-\tau)\, y(\tau)\, d\tau$.

Variant 28

1. Find image of the original $f(t)=(t^2+1)e^{-t}$.

2. Find original of the following image $F(p)=\dfrac{2p+3}{p^3+1}$.

3. Solve the Cauchy problem by operational calculations $y'+y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}1, & 0\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y'=\dfrac{1}{e^t+1},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t-\int_0^t (t-\tau)y(\tau)d\tau$.

Variant 29

1. Find image of the original $f(t)=te^{2t}cht$.

2. Find original of the following image $F(p)=\dfrac{2pe^{-p}}{p^2-4}$.

3. Solve the Cauchy problem by operational calculations $y'+4y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}1, & 0\le t<3\\ 2, & t\ge 3\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''+y'=\dfrac{e^t}{e^t+1},\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=t+\int_0^t \sin(t-\tau)y(\tau)d\tau$.

Variant 30

1. Find image of the original $f(t)=\int_0^t e^{\tau}\sin\tau d\tau$.

2. Find original of the following image $F(p)=\dfrac{1}{p^3+2p^2+p}$.

3. Solve the Cauchy problem by operational calculations $y'+y=f(t),\ y(0)=0$ де $f(t)=\begin{cases}1, & 0\le t<2\\ 0, & t\ge 2\end{cases}$.

4. Using the integral Duhamel solve the Cauchy problem $y''-y=tht,\ y(0)=y'(0)=0$.

5. Solve the integral equation $y(t)=e^t+\int_0^t e^{t-\tau}y(\tau)d\tau$.

Answer:

Variant 1

1. $F(p)=\dfrac{1}{2}\ln\dfrac{p+2}{\sqrt{(p+2)^2+4}}$
2. $f(t)=2e^{2t}+5te^{2t}$
3. $y(t)=\dfrac{1}{4}\left(t-\dfrac{1}{2}\sin 2t\right)\eta(t)-\dfrac{1}{2}\left((t-1)-\dfrac{1}{2}\sin 2(t-1)\eta(t-1)\right)+\dfrac{1}{4}\left(t-2-\dfrac{1}{2}\sin 2(t-2)\right)\eta(t-2)$
4. $y(t)=-sht+2cht\cdot arctg\left(th\dfrac{t}{2}\right)$
5. $y(t)=\eta(t)$

Variant 2

1. $F(p)=\dfrac{e^{-5p}(p-4)}{(p-4)^2+1}$
2. $f(t)=3ch2t$
3. $y(t)=\eta(t)\left(1-e^{-t}-te^{-t}\right)-\eta(t-1)\left(1-e^{-(t-1)}(t-1)e^{-(t-1)}\right)$

4. $y(t)=e^{t}\left(t\cdot arctgt-\frac{1}{2}\ln\left(1+t^{2}\right)\right)$

5. $y(t)=\cos 2t-\frac{1}{2}\sin 2t$

Variant 3

1. $F(p)=\frac{10p}{\left(p^{2}+25\right)^{2}}$

2. $f(t)=te^{t}+\frac{1}{2}t^{2}$

3. $y(t)=\frac{1}{2}\left(1-e^{-t}(\cos t+\sin t)\right)\eta(t)-\frac{1}{2}\left(1-e^{-(t-2)}\left(\cos(t-2)+\sin(t-2)\right)\right)\eta(t-2)$

4. $y(t)=e^{t}-1-\left(1+e^{t}\right)(t+\ln 2)+\left(1+e^{t}\right)\ln\left(1+e^{t}\right)$

5. $y(t)=\eta(t)$

Variant 4

1. $F(p)=\frac{1}{2}\left(\frac{p+4}{(p+4)^{2}+9}+\frac{p+4}{(p+4)^{2}+49}\right)$

2. $f(t)=\frac{1}{5}\left(\frac{1}{2}\sin 2t-\frac{1}{3}\sin 3t\right)$

3. $y(t)=\frac{3}{4}\left(1-e^{-4t}\right)\eta(t)+\frac{3}{4}\left(1-e^{-4(t-3)}\right)\eta(t-3)$

4. $y(t)=te^{t}\sin t$

5. $y(t)=\frac{3}{2}\cos 2t+\frac{1}{2}$

Variant 5

1. $F(p)=\frac{1}{2}\ln\frac{p^{2}+4}{p^{2}+36}$

2. $f(t)=\cos 4(t-2)\eta(t-2)$

3. $y(t)=\left(1-e^{-t}\right)\eta(t)-\left(1-e^{-(t-2)}\right)\eta(t-2)$

4. $y(t) = -2sht\left(arctge^t - \frac{\pi}{4}\right) + cht - 2$
5. $y(t) = \frac{3}{2}\cos 2t + \frac{1}{2}$

Variant 6

1. $F(p) = \frac{1}{2}\ln\frac{\sqrt{p^2+4}}{p}$
2. $f(t) = -\frac{1}{3}e^t + \frac{1}{4}e^{2t} + \frac{1}{12}e^{-2t}$
3. $y(t) = \frac{2}{3}\left(1 - e^{-3t}\right)\eta(t) - \frac{1}{3}\left(1 - e^{-3(t-1)}\right)\eta(t-1)$
4. $y(t) = tsht - cht\ln cht$
5. $y(t) = 1 + 2t + \frac{t^2}{2} + \frac{t^3}{3}$

Variant 7

1. $F(p) = -\frac{1}{4}\ln\frac{p^2+16}{p^2+100}$
2. $f(t) = e^{-2t} + e^{-t}(t-1)$
3. $y(t) = 3e^{-2t}\eta(t) + \left(1 - e^{-2t}\right)\eta(t) - \frac{1}{2}\left(1 - e^{-2(t-1)}\right)\eta(t-1)$
4. $y(t) = te^t - \left(1 + e^t\right)\ln\frac{1+e^t}{2}$
5. $y(t) = \frac{1}{3}\left(e^t - e^{\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t + \sqrt{3}e^{\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t\right)$

Variant 8

1. $F(p) = \frac{1}{p\left(p^2+1\right)}$
2. $f(t) = \frac{1}{12}e^{2t} - \frac{1}{12}e^{-t}\cos\sqrt{3}t - \frac{1}{4\sqrt{3}}e^{-t}\sin\sqrt{3}t$

3. $y(t)=(1-\cos t)\eta(t)-2(1-\cos(t-1)\eta(t-1))+(1-\cos(t-2))\eta(t-2)$
4. $y(t)=e^{t}((t+1)\ln(t+1)-t)$
5. $y(t)=t+\frac{1}{6}t^{3}$

Variant 9

1. $F(p)=\frac{1}{2}\left(\frac{p-2}{(p-2)^{2}+9}-\frac{p+2}{(p+2)^{2}+9}\right)$
2. $f(t)=1-2e^{t}+e^{3t}$
3. $y(t)=\left(1-e^{-t}\right)\eta(t)-\left(1-e^{-(t-2)}\right)\eta(t-2)$
4. $y(t)=\frac{1}{3}\left(e^{t}-1\right)-\frac{t}{9}e^{t}+\frac{1}{9}e^{t}\ln\frac{e^{t}+3}{4}$
5. $y(t)=\frac{2}{5}e^{2t}+\frac{3}{5}\cos t+\frac{1}{5}\sin t$

Variant 10

1. $F(p)=\frac{1}{4}\left(\frac{p}{p^{2}+9}+\frac{3p}{p^{2}+1}\right)$
2. $f(t)=\frac{1}{12}(3-4\cos t+\cos 2t)$
3. $y(t)=2\left(\sin^{2}\frac{t}{2}\eta(t)-2\sin^{2}\frac{t-1}{2}\eta(t-1)+\sin^{2}\frac{t-2}{2}\eta(t-2)\right)$
4. $y(t)=sht\left(t-\frac{2}{e^{t}+1}+1\right)-cht\cdot\ln\frac{1+cht}{2}$
5. $y(t)=2+t-e^{\frac{t}{2}}\left(\cos\frac{\sqrt{3}}{2}t-\sqrt{3}\sin\frac{\sqrt{3}}{2}t\right)$

Variant 11

1. $F(p)=\ln\left|\frac{p+5}{p+1}\right|$

2. $f(t)=\frac{t}{2\sqrt{2}}\sin\left(t\sqrt{2}\right)$
3. $y(t)=2\left(t-e^{-t}-1\right)\eta(t)-\left(t-1+e^{-(t-1)}-1\right)\eta(t-1)-\left(t-2+e^{-(t-2)}-1\right)\eta(t-2)$
4. $y(t)=\frac{1}{2}\left(e^{t}-1-te^{t}\right)+sht\cdot\ln\frac{1+e^{t}}{2}$
5. $y(t)=-\frac{1}{16}-\frac{t}{8}+\frac{3t^2}{8}-\frac{t^3}{12}+\frac{1}{16}e^{2t}$

Variant 12

1. $F(p)=\frac{arcctg\ p}{p}$
2. $f(t)=e^{(t-3)}\eta(t-3)-\eta(t-3)$
3. $y(t)=(1-\cos t)\eta(t)-\left(t-\sin t)\eta(t)\right)+\left(t-1-\sin(t-1)\right)\eta(t-1)$
4. $y(t)=\frac{sh^2 2t}{ch2t}$
5. $y(t)=\frac{1}{2}e^{-t}+\frac{1}{6}e^{t}+\frac{1}{3}e^{\frac{-t}{2}}\left(\cos\frac{\sqrt{3}}{2}t-\sqrt{3}\sin\frac{\sqrt{3}}{2}t\right)$

Variant 13

1. $F(p)=\frac{5}{2}\left(\frac{1}{(p-2)^2+25}-\frac{1}{(p+2)^2+25}\right)$
2. $f(t)=\frac{1}{3}\left(\frac{3}{4}-\cos t+\frac{1}{4}\cos 2t\right)$
3. $y(t)=\frac{1}{4}\left(t-\frac{1}{2}\sin 2t\right)\eta(t)-\frac{1}{4}\left(t-1-\frac{1}{2}\sin 2(t-1)\right)\eta(t-1)-\frac{1}{4}\left(1-\cos 2(t-1)\right)\eta(t-1)$
4. $y(t)=2sht\cdot arctge^{t}-1+cht$
5. $y(t)=\frac{1}{3}\left(e^{t}-e^{-t}+\frac{\sqrt{2}}{2}\sin\left(t\sqrt{2\sqrt{\ }}\right)\right)$

Variant 14

1. $F(p)=\frac{1}{2}\left(\frac{1}{(p+1)^2+1}+\frac{3}{(p+1)^2+9}\right)$
2. $f(t)=\frac{3}{4!}t^4e^{5t}+\frac{16}{5!}t^5e^{5t}$
3. $y(t)=\left(e^t-1-t\right)\eta(t)+\left(e^{t-1}-1-(t-1)\right)\eta(t-1)-2\left(e^{t-2}-1-(t-2)\right)\eta(t-2)$
4. $y(t)=\frac{e^t+1}{e^t}\ln\frac{e^t+1}{2}+\frac{1-e^t}{e^t}$
5. $y(t)=te^t$

Variant 15

1. $F(p)=\frac{8!}{(p+3)^9}$
2. $f(t)=\frac{5t}{6}\sin 3t$
3. $y(t)=\left(\frac{1}{16}e^{4t}-\frac{1}{16}-\frac{t}{4}\right)\eta(t)-2\left(\frac{1}{16}e^{4(t-1)}-\frac{1}{16}-\frac{t-1}{16}\right)\eta(t-1)+\left(\frac{1}{16}e^{4(t-2)}-\frac{1}{16}-\frac{t-2}{16}\right)\eta(t-2)$
4. $y(t)=t^2e^{-t}\,\frac{t+2}{(t+1)^2}+e^{-t}\left(t+1-\ln(t+1)\right)$
5. $y(t)=\frac{1}{3}\left(4-\cos\left(t\sqrt{3}\right)\right)$

Variant 16

1. $F(p)=\frac{e^{-5p}(p-4)}{(p-4)^2+1}$
2. $f(t)=-4e^{2t}+5e^{3t}$
3. $y(t)=(1-\cos t)\eta(t)-2\left(1-\cos(t-1)\eta(t-1)\right)+\left(1-\cos(t-2)\right)\eta(t-2)$
4. $y(t)=\frac{1}{2}\frac{sh^2 2t}{cht}$
5. $y(t)=cht-te^{-t}$

Variant 17

1. $F(p)=\frac{3}{2}\left(\frac{1}{(p-2)^2+9}+\frac{1}{(p+2)^2+9}\right)$
2. $f(t)=\frac{1}{8}(\cos t-\cos 3t)$
3. $y(t)=\frac{1}{3}\left(1-\cos\sqrt{3}t\right)\eta(t)-\frac{1}{3}\left(1-\cos\sqrt{3}(t-4)\eta(t-4)\right)$
4. $y(t)=\frac{1}{2}\left(e^t-1\right)-\ln\frac{1+e^t}{2}$
5. $y(t)=\frac{1}{2}(cht+\cos t)$

Variant 18

1. $F(p)=\frac{8p}{\left(p^2+16\right)^2}$
2. $f(t)=(t-2)e^{t-2}\eta(t-2)$
3. $y(t)=(1-\cos t)\eta(t)+3\left(1-\cos(t-2)\right)\eta(t-2)$
4. $y(t)=e^{-t}(\frac{t^2}{2}+t-(t+1)\ln(t+1))$
5. $y(t)=t-\frac{t^3}{6}$

Variant 19

1. $F(p)=-\frac{e^{-p\pi}}{p^2+1}$
2. $y(t)=\frac{1}{3}\left(\frac{sh2t}{2}-sht\right)$
3. $y(t)=\frac{1}{4}(1-\cos 2t)\eta(t)-\frac{1}{4}\left(1-\cos 2(t-\pi)\right)\eta(t-\pi)$
4. $y(t)=\left(e^t+2\right)\ln\frac{2+e^t}{3}-e^t+1$
5. $y(t)=\frac{2}{\sqrt{5}}e^{-\frac{t}{2}}sh\frac{\sqrt{5}}{2}t$

Variant 20

1. $F(p)=\frac{1}{2}\left(\frac{p-1}{(p-1)^2+9}-\frac{p+1}{(p+1)^2+9}\right)$
2. $f(t)=1-2e^t+e^{3t}$
3. $y(t)=\frac{1}{9}\eta(t-3)-\frac{1}{9}\cos 3(t-3)\eta(t-3)$
4. $y(t)=\frac{1}{cht}-1+e^t\left(tht-\ln\frac{2e^{2t}}{e^{2t}+1}-\frac{1}{e^{2t}+1}+\frac{1}{2}\right)$
5. $y(t)=\frac{1}{2}sht+\frac{1}{2}\sin t$

Variant 21

1. $F(p)=\frac{2}{p\left(p^2+4\right)}$
2. $y(t)=e^t-1-t-\frac{t^2}{2}$
3. $y(t)=\frac{1}{2}\eta(t-2)-\frac{1}{2}\cos 2(t-2)\eta(t-2)$
4. $y(t)=\frac{cht}{1+e^t}-e^{-t}\left(\frac{1}{1+e^t}+\ln\left(1+e^t\right)\right)$
5. $y(t)=t-\frac{t^2}{2}$

Variant 22

1. $F(p)=\frac{2\left(p^2-9\right)}{\left(p^2+9\right)^2}+\frac{p}{p^2+9}$
2. $y(t)=1-\cos t$
3. $y(t)=\left(1-e^{-2t}\right)\eta(t)-\frac{1}{2}\left(1-e^{-2(t-1)}\right)\eta(t-1)$
4. $y(t)=e^{-t}(arctgt-\ln\left(t^2+1\right))$
5. $y(t)=1-\frac{t^2}{2}$

Variant 23

1. $F(p)=\frac{1}{2}\left(\frac{1}{p^2}+\frac{1}{(p+2)^2}\right)$
2. $f(t)=\frac{1}{2}(\sin t+t\cos t)$
3. $y(t)=\frac{1}{9}(e^{-3(t-2)}+3(t-2)-1)\eta(t-2)$
4. $y(t)=e^{t}\ln cht$
5. $y(t)=t-\frac{t^3}{6}$

Variant 24

1. $F(p)=\frac{4p}{(p^2+1)^2}+\frac{2}{p^2+4}$
2. $f(t)=\frac{1}{2}(t-3)^2 e^{-(t-3)}\eta(t-3)$
3. $y(t)=\frac{1}{9}(e^{-3(t-1)}+3(t-1)-1)\eta(t-1)$
4. $y(t)=e^{-t}\ln cht$
5. $y(t)=1+2t+\frac{t^2}{2}+\frac{t^3}{3}$

Variant 25

1. $F(p)=\frac{2}{(p-3)^3}$
2. $f(t)=\frac{2}{3\sqrt{3}}e^{\frac{t}{2}}\sin\frac{3\sqrt{3}}{2}t$
3. $y(t)=e^{-3t}\eta(t)+\frac{1}{3}\left(t-e^{-3t}\right)\eta(t)-\frac{2}{3}\left((t-1)-e^{-3(t-1)}\right)\eta(t-1)$
4. $y(t)=e^{-t}\left((t+1)\ln(t+1)-t\right)$
5. $y(t)=\frac{1}{2}tcht+\frac{1}{2}sht$

Variant 26

1. $F(p)=\frac{1}{2}\left(\frac{1}{p^2}+\frac{p^2-4}{\left(p^2+4\right)^2}\right)$
2. $f(t)=\sin 3(t-4)\eta(t-4)$
3. $y(t)=\left(\frac{1}{2}-\frac{1}{4}\cos 2(t-2)\right)\eta(t-2)$
4. $y(t)=\frac{1}{3}\left(e^t-1\right)-\frac{t}{9}e^t+\frac{1}{9}e^t\ln\frac{e^t+3}{4}$
5. $y(t)=1-2te^t$

Variant 27

1. $F(p)=\frac{2}{p^3}\cdot\frac{1}{p^2-1}$
2. $f(t)=(t-2)\eta(t-2)$
3. $y(t)=\left(1-e^{-2t}\right)\eta(t)-\frac{1}{2}\left(1-e^{-2(t-1)}\right)\eta(t-1)$
4. $y(t)=\frac{1}{2}\left(e^t-1-te^t\right)+sht\cdot\ln\frac{e^t+1}{2}$
5. $y(t)=1+t+\frac{t^2}{2}+\frac{t^3}{6}$

Variant 28

1. $F(p)=\frac{2}{(p+1)^3}+\frac{1}{p+1}$
2. $f(t)=\frac{1}{3}e^{-t}-\frac{1}{3}e^{\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t-\frac{5}{\sqrt{3}}e^{\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t$
3. $y(t)=\left(1-e^{-t}\right)\eta(t)-\left(1-e^{-(t-2)}\right)\eta(t-2)$
4. $y(t)=e^t-1-\left(1+e^t\right)\left(t+\ln 2\right)+\left(1+e^t\right)\ln\left(1+e^t\right)$
5. $y(t)=\sin t$

Variant 29

1. $F(p)=\frac{1}{2}\left(\frac{1}{(p-3)^2}+\frac{1}{(p-1)^2}\right)$
2. $f(t)=(ch2(t-1))\eta(t-1)$
3. $y(t)=\frac{1}{4}\left(t-e^{-4t}\right)\eta(t)+\frac{1}{4}\left((t-3)-e^{-4(t-3)}\right)\eta(t-3)$
4. $y(t)=\left(e^{-t}+1\right)\ln\frac{e^{t}+1}{2}+e^{-t}-1$
5. $y(t)=1+\frac{t^2}{2}$

Variant 30

1. $F(p)=\frac{1}{p^2(p-2)}$
2. $f(t)=1-te^{-t}-e^{-t}$
3. $y(t)=\left(1-e^{-t}\right)\eta(t)-\left(1-e^{-(t-2)}\right)\eta(t-2)$
4. $y(t)=-sht+2cht\cdot arctg\left(th\frac{t}{2}\right)$
5. $y(t)=e^{2t}$

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